ANALYSIS IN SEVERAL COMPLEX VARIABLES: SUBHARMONIC AND PSH FUNCTIONS

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Exercice 1. Let Δ be the unit polydisk in $\mathbb{C}$. Pick any sequence of distinct points $z_n \neq 0$ such that $\{z_n\}$ is dense in Δ.

(a) Construct a sequence of positive number $\alpha_n > 0$ such that $\sum_n \alpha_n \log |z_n| > -\infty$.

(b) Prove that the sequence $u_n(z) = \sum_{j \leq n} \alpha_j \log(\frac{1}{2}|z - z_j|)$ is decreasing to a subharmonic function u on Δ.

(c) Prove that $\{u = -\infty\}$ is dense (consider sublevel sets $\{u < -N\}$ and use Baire theorem).

Exercice 2. Prove that a function $u \colon \Omega \to [-\infty, \infty)$ such that u and $-u$ are both subharmonic is harmonic.

Exercice 3. Let $u_n \colon \Omega \to [-\infty, \infty)$ be any sequence of subharmonic functions such that $u_n \leq 0$. Define the functions $u(z) = \sup_n u_n(z)$, and

$$u^*(z) = \limsup_{w \to z} u(w) \ .$$

Prove that u^* is subharmonic.

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