SOME RECENT RESULTS ON LINEAR CONFIGURATIONS IN SUBSETS OF CYCLIC GROUPS

PABLO CANDELA

A version of Roth's theorem on 3-term progressions states the following: for any $\alpha > 0$ and positive integer N, let $m(\alpha, N)$ denote the minimum, over all subsets A of $\mathbb{Z}_N$ of size at least αN, of the normalized count of 3-term arithmetic progressions in A, that is $N^{-2} \sum_{x,r \in \mathbb{Z}_N} 1_A(x) 1_A(x+r) 1_A(x+2r)$. Then there exists $c(\alpha) > 0$ such that $m(\alpha, N) \geq c$ uniformly for all N.

It is a well-known problem to improve estimates for $c(\alpha)$. But does $m(\alpha, N)$ even converge as N increases? Croot showed that it does provided N is restricted to prime values.

I will discuss recent joint work with Olof Sisask towards extending this result of Croot, and other related convergence results, to linear configurations other than 3-term progressions (i.e. to solutions of a given system of integer linear-equations other than $x - 2y + z = 0$). Via these results, discrete problems such as improving the bounds in Roth's theorem are connected with similar problems in a continuous setting that can be viewed as limit versions of the discrete ones.