KRULL MONOIDS AND ZERO-SUM THEORY

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A monoid H is called a Krull monoid if there is a divisibility preserving homomorphism into a factorial monoid. Let H be a Krull monoid with finite class group G and suppose that every class contains a prime divisor (all this holds true for rings of integers in algebraic number fields). Then every non-unit $a \in H$ can be written as a product of irreducible elements, say $a = \prod_{i=1}^{k} u_i$. The number of irreducible factors is called the length of the factorization, and $\mathsf{L}(a) = \{k \in \mathbb{N} \mid a \text{ has a factorization of length } k\}$ denotes the *set of lengths* of a. Here are some simple observations:

- H is a factorial monoid if and only if $|G| = 1$.
- (Carlitz 1960) $|G| \leq 2$ if and only if $|\mathsf{L}(a)| = 1$ for all non-units $a \in H$.
- If $|G| \geq 3$, then all sets of lengths are finite, and
  for every $m \in \mathbb{N}$, there is an $a_m \in H$ with $|\mathsf{L}(a_m)| = m$.

Sets of lengths in H can be studied in the monoid of zero-sum sequences $\mathcal{B}(G)$ associated to H. A zero-sum sequence means a finite unordered sequence of group elements from G which sums up to zero (the repetition of elements is allowed). The set of zero-sum sequences forms a monoid where the operation is just the juxtaposition of sequences. Moreover, the monoid $\mathcal{B}(G)$ is a Krull monoid again, and it can be studied with methods from Additive Combinatorics.

In the first part of my talk, I highlight this well-known relationship between general Krull monoids and monoids of zero-sum sequences. In the second part of my talk, I will outline some recent joint work with A. Geroldinger and F. Kainrath. We could show that there is a similar relationship between certain non-Krull monoids (including certain non-principal orders in number fields) and zero-sum sequences.

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