TRIANGULABILITY AND IRREGULARITY

by

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Abstract. In this note, I answer a question of Jean-Baptiste Teyssier concerning the triangulability of the real blowing up map in a way compatible with the Stokes structure of a meromorphic flat bundle.

1. Introduction. Let (X, D) be a smooth complex manifold with a normal crossing divisor, let $\varpi : \widetilde{X} \to X$ be the real blowing up of the smooth local irreducible components of D and let $(\mathcal{L}, \mathcal{L}_\bullet)$ be a good Stokes filtered local system on (X, D) (e.g. associated with a good meromorphic flat bundle on (X, D) by the de Rham functor). For each stratum E of the natural stratification of D, consider the restriction $\varpi_{|E} : \varpi^{-1}(E) \to E$. Corollary 14 shows that this map can be stratified in a way compatible with the Stokes structure of $(\mathcal{L}, \mathcal{L}_\bullet)$ so that each pair of adjacent strata satisfy Whitney conditions and Thom condition. It follows from **[4, 5]** that it is triangulable in a way compatible with the Stokes structure.

2. Preliminaries on stratifications.

Definition 1 **(Stratification).**

(1) Let Y be a topological space. A *stratification* $\mathcal{S}$ of Y is a locally finite family of locally closed subsets S of Y, called strata, which form a partition of Y and such that the closure of each stratum is equal to a union of strata. (We do not assume that the strata are connected.) For strata S', S in $\mathcal{S}$, we denote $S' \prec S$ the property $S' \subset \overline{S}$.

(2) Let $\mathcal{F}$ be a locally finite family of closed subsets of Y and let $\overline{\mathcal{F}}$ denote the locally finite family of locally closed subsets obtained from $\mathcal{F}$ by iterating a finite number of intersections and complements. We say that a stratification $\mathcal{S}$ of Y is *compatible* with $\mathcal{F}$, or that $\mathcal{S}$ is a stratification of $(Y, \mathcal{F})$, if any locally closed subset of $\overline{\mathcal{F}}$ is a union of strata in $\mathcal{S}$.

(3) Let $f : X \to Y$ be a continuous map and let $\mathcal{S}, \mathcal{T}$ be stratifications of X and Y respectively. We say that $(\mathcal{S}, \mathcal{T})$ is a stratification of f if f maps each stratum in $\mathcal{S}$ to a stratum in $\mathcal{T}$, called its image stratum.

Below, we work within the subanalytic category, where topological spaces and continuous maps are assumed to be subanalytic, and strata are real-analytic manifolds. The restriction of a stratified map to a stratum S is assumed to be real-analytic and a submersion to its image stratum T; in particular, $f(S)$ is open in T. A stratification $(\mathcal{S}, \mathcal{T})$ of $f : X \to Y$ is a Whitney regular Thom stratification if each pair of adjacent strata in $\mathcal{S}$ or in $\mathcal{T}$ satisfies Whitney's conditions (a) and (b) and each pair of adjacent strata in $\mathcal{S}$ satisfies Thom's condition (a_f). As we will have to apply base change to stratified maps, and as Whitney's conditions at the source do not necessarily behave well by base change (although Thom's condition does, see [**3**]), we will consider Verdier's (w) condition (see [**6**]) and its relative analogue (w_f) (see [**2, 1**]). We refer to [**1**, Def. 1.1] for the definition of (w_f), and Verdier's (w) condition is obtained by applying (w_f) to the constant map f.

Recall the following well-known properties:

(i) (w) implies Whitney's (a) and (b), and (w_f) implies Thom's (a_f),
(ii) any subanalytic stratification of a subanalytic set can be refined into a (w)-regular stratification,
(iii) any subanalytic stratification $(\mathcal{S}, \mathcal{T})$ of a continuous subanalytic map $f : X \to Y$ can be refined into a subanalytic stratification $(\mathcal{S}', \mathcal{T}')$ of f such that both $\mathcal{S}$ and $\mathcal{T}$ are (w)-regular.

Definition 2. Assume that X is closed subanalytic in $\mathbb{R}^\ell \times \mathbb{R}^p$, that Y is closed subanalytic in $\mathbb{R}^p$ and f is the second projection. We say that a subanalytic stratification $(\mathcal{S}, \mathcal{T})$ of f is (w)-regular if $\mathcal{T}$ satisfies (w) and $\mathcal{S}$ satisfies (w_f).

The interest of this definition comes from the following two lemmas.

Lemma 3. *Let $f : X \to Y$ be as in Definition 2 and let $(\mathcal{S}, \mathcal{T})$ be a subanalytic stratification of f. If $(\mathcal{S}, \mathcal{T})$ is (w)-regular, then $\mathcal{S}$ satisfies (w).*

Proof. The regularity conditions are local and can be expressed by using any local embedding into real manifolds. We can thus assume that f induced by the projection $\mathbb{R}^m = \mathbb{R}^\ell \times \mathbb{R}^p \to \mathbb{R}^p$. We use the standard scalar products to define distances d. For nonzero subspaces E', E of $\mathbb{R}^n$ $(n = p, m)$, we set

$$\delta(E', E) = \sup_{\substack{u' \in E' \\ u \in E^\perp}} \frac{|\langle u', u\rangle|}{\|u'\|\,\|u\|},$$

so that $\delta(E', E) = 0 \iff E' \subset E$. Given a pair of adjacent strata $S' \prec S$ and $x' \in S'$, (S, S') satisfies (w) resp. (w_f) if there exists a neighbourhood of x' and a constant $C > 0$ such that, for any $x \in S$ in this neighbourhood,

$$\delta(T_{x'}S', T_xS) \leqslant Cd(x', x), \quad \text{resp. } \delta\big(T_{x'}f^{-1}_{|S'}(f(x')), T_xf^{-1}_{|S}(f(x))\big) \leqslant Cd(x', x).$$

By the submersion property, we have in $\mathbb{R}^p \times \mathbb{R}^m$:

$$T_xS = \big(T_xf^{-1}_{|S}(f(x))\big) \times \big(T_{f(x)}T\big),$$

and similarly for $T_{x'}S'$. The conclusion follows from the property that, given nonzero subspaces E, E' of $\mathbb{R}^\ell$ and F, F' of $\mathbb{R}^p$,

$$\delta(E' \times F', E \times F) \leqslant \delta(E', E) + \delta(F', F). \qquad \square$$

Let $f : X \to Y$ be as in Definition 2 and let $(\mathcal{S}, \mathcal{T})$ be a subanalytic stratification of f. A subanalytic refinement $\mathcal{T}'$ of $\mathcal{T}$ induces a refinement $\mathcal{S}'$ of $\mathcal{S}$ so that $(\mathcal{S}', \mathcal{T}')$ remains a subanalytic stratification of f in the following way: assume that a stratum T in $\mathcal{T}$ is partitioned into strata T' of $\mathcal{T}'$; for any stratum S in $\mathcal{S}$ such that $f(S)$ is contained in T (as an open subset), we consider the partition of S by the smooth subanalytic submanifolds $f^{-1}_{|S}(T')$; one checks that this partition $\mathcal{S}'$ is a stratification (i.e., the closure of any stratum is a union of strata). We say that $(\mathcal{S}', \mathcal{T}')$ is the refinement of $(\mathcal{S}, \mathcal{T})$ induced by $\mathcal{T}'$.

Let $g : Z \to Y$ be a continuous subanalytic map and assume that there exists a stratification $\mathcal{U}$ of Z such that $(\mathcal{U}, \mathcal{T})$ is a stratification of g. The fiber product $Z \times_Y X$ is partitioned by the fiber products $U \times_T S$ for pairs (U, S) of strata mapping submersively to the same stratum T in Y. One checks that $(\mathcal{U} \times_{\mathcal{T}} \mathcal{S}, \mathcal{U})$ is a subanalytic stratification of the base-changed map $Z \times_Y X \to Z$. In practice, given $\mathcal{T}$, $\mathcal{U}$ may not exist. However, according to (iii) above, we can refine $\mathcal{T}$ as a stratification $\mathcal{T}'$ so that there exists $\mathcal{U}$ such that $(\mathcal{U}, \mathcal{T}')$ is a stratification of g.

Lemma 4. *Let $f : X \to Y$ be as in Definition 2 and let $(\mathcal{S}, \mathcal{T})$ be a (w)-regular subanalytic stratification of f.*

1. *(Refinement) Let $\mathcal{T}'$ be a (w)-regular refinement of $\mathcal{T}$ and let $(\mathcal{S}', \mathcal{T}')$ be the induced refinement of $(\mathcal{S}, \mathcal{T})$. Then $(\mathcal{S}', \mathcal{T}')$ is (w)-regular.*
2. *(Base change) Let $g : Z \to Y$ be a subanalytic morphism and let $\mathcal{U}$ be a (w)-regular subanalytic stratification of Z such that $(\mathcal{U}, \mathcal{T})$ is a subanalytic stratification of g (we do not assume that this pair is (w)-regular). Then $(\mathcal{U} \times_{\mathcal{T}} \mathcal{S}, \mathcal{U})$ is a (w)-regular subanalytic stratification of $Z \times_Y X \to Z$.*

Sketch of proof. In both cases, the tangent space to the fiber of a stratum remains unchanged after refinement or base change. The change of behaviour of the function δ is only due to the change in the stratification $\mathcal{T}$ and the assumptions imply that (w) remains satisfied for $\mathcal{T}'$ or $\mathcal{U}$. $\square$

3. Linear stratifications.

***Definition 5* (Linear stratification of a linear map).** Let $f:\mathbb{R}^m\to\mathbb{R}^p$ be a linear map. A *linear stratification* of f consists of a stratification $(\mathcal{S}_m,\mathcal{S}_p)$ of f such that the closure of each stratum is a linear affine subspace of $\mathbb{R}^m$, resp. $\mathbb{R}^p$.

Let us notice some obvious properties of a linear stratification of a linear map. Given a stratum S_m, its tangent space T_xS_m at $x\in S_m$ is independent of x and equal to the direction of $\overline{S}_m$. The boundary $\partial S_m=\overline{S}_m\smallsetminus S_m$ is a union of strata, hence the union of their closures. It is thus a union of linear affine subspaces. For a stratum $S\subset\partial S_m$, the rank of f on $\overline{S}$ is smaller than or equal to the rank of f on $\overline{S}_m$. The closure $\overline{f(S_m)}$ is equal to the linear affine subspace $f(\overline{S}_m)$ and $f(S_m)=f(\overline{S}_m)\smallsetminus\bigcup_S f(\overline{S})$, where S varies in the set of strata of ∂S_m on which the rank of f is strictly smaller than the rank of f on S_m.

Lemma 6. *Let $\mathcal{F}$ be a locally finite family of linear affine subspaces of $\mathbb{R}^m$. Then there exists a linear stratification $\mathcal{S}_m$ of $\mathbb{R}^m$ which is compatible with $\mathcal{F}$.*

Proof. We first replace $\mathcal{F}$ with a larger locally finite family by adding to it all iterated intersections of the linear affine spaces in $\mathcal{F}$. In such a way, we can assume that $\mathcal{F}$ is stable by intersections. We then define the stratification as follows. A stratum of dimension $d\geqslant 0$ is the complement, in a linear affine space of dimension d belonging to $\mathcal{F}$, of its subspaces belonging to $\mathcal{F}$ of dimension $<d$. One checks by induction on d that any space of dimension d in $\mathcal{F}$ is a union of strata, and the closure of a stratum of dimension d is a linear affine space of dimension d. □

Lemma 7. *Let $f:\mathbb{R}^m\to\mathbb{R}^p$ be a linear map and let $\mathcal{F}$ be a locally finite family of linear affine subspaces of $\mathbb{R}^m$ stable by intersections such that the family $f(\mathcal{F})$ is locally finite in $\mathbb{R}^p$. Then there exists a linear stratification $(\mathcal{S}_m,\mathcal{S}_p)$ of f such that $\mathcal{S}_m$ is compatible with $\mathcal{F}$.*

Proof. Let $\mathcal{G}$ by the family obtained from $f(\mathcal{F})$ by adding all intersections. It remains locally finite. Any F in $\mathcal{F}$ has image equal to some G in $\mathcal{G}$, hence is contained in $f^{-1}(G)$. Let $\mathcal{F}'$ be the (locally finite) family stable by intersections generated by $\mathcal{F}\cup f^{-1}\mathcal{G}$. For any F' in $\mathcal{F}'$, $f(F')$ belongs to $\mathcal{G}$: since $\mathcal{F}$ and $f^{-1}\mathcal{G}$ are stable by intersections, we can write $F'=F\cap f^{-1}(G)$ for some F in $\mathcal{F}$ and G in $\mathcal{G}$; then $G'=f(F)\cap G$ also belongs to $\mathcal{G}$ and we have $F'=F\cap f^{-1}(G')$, so that $f(F')=G'$.

The linear stratifications $\mathcal{S}_m$ resp. $\mathcal{S}_p$ associated with $\mathcal{F}'$ resp. $\mathcal{G}$ as in Lemma 6 form a linear stratification of f. □

Lemma 8. *Every linear stratification $(\mathcal{S}_m,\mathcal{S}_p)$ of a linear map $f:\mathbb{R}^m\to\mathbb{R}^p$ is (w)-regular.*

Proof. Let us check (w_f) for example. Let $S\subset\overline{S}_m$ be a pair of strata (i.e., $\overline{S}$ is a linear affine subspace of $\overline{S}_m$). Let $x\in S$ and $y\in S_m$ in some neighbourhood of x. The tangent space to $f_{|S_m}$ at y is equal to $\ker[f : T_yS_m \to \mathbb{R}^p]$, that is, $\ker[f : \overline{S}_m \to \mathbb{R}^p]$ and does not depend on y. Similarly, the tangent space of $f_{|S}$ at x is equal to $\ker[f : \overline{S} \to \mathbb{R}^p]$. Since $\overline{S} \subset \overline{S}_m$, the latter is included into the former and the distance $d(T_xS_{f(x)}, T_yS_{m,f(y)})$ is zero, hence smaller that $d(x, y)$. □

4. An example. Let $L_1, \dots, L_p$ be a finite family of linear forms $\mathbb{R}^\ell \to \mathbb{R}$ with integer coefficients (coordinates $\boldsymbol{\theta} = (\theta_1, \dots, \theta_\ell)$ on $\mathbb{R}^\ell$). We consider the family of linear forms $\Lambda_j(\boldsymbol{\theta}, \boldsymbol{\eta}) = L_j(\boldsymbol{\theta}) + \eta_j$ $(j = 1, \dots, p)$, where $\boldsymbol{\eta} = (\eta_1, \dots, \eta_p)$ are the coordinates on $\mathbb{R}^p$. We consider the family $\mathcal{F}'$ of affine hyperplanes of $\mathbb{R}^m = \mathbb{R}^\ell \times \mathbb{R}^p$ defined by

$$\Lambda_j(\boldsymbol{\theta}, \boldsymbol{\eta}) = \pm\pi/2 + 2k\pi, \quad j = 1, \dots, p,\ k \ni \mathbb{Z}.$$

Let $\boldsymbol{d} = (d_1, \dots, d_\ell)$ be a multi-index of positive integers and let us denote by $\frac{2\pi}{\boldsymbol{d}}\mathbb{Z}^\ell$ the lattice $\prod_{i=1}^{\ell}(\frac{2\pi}{d_i}\mathbb{Z})$ of $\mathbb{R}^\ell$. We denote by $\mathcal{F}$ the locally finite family of affine hyperplanes having equation

$$\Lambda_j(\boldsymbol{\theta} + 2\boldsymbol{k}\pi/\boldsymbol{d}, \boldsymbol{\eta}) = \pm\pi/2 + 2k\pi, \quad j = 1, \dots, p,\ \boldsymbol{k} \in \mathbb{Z}^\ell,\ k \ni \mathbb{Z}.$$

By construction, this locally finite family is stable by the additive action of $\frac{2\pi}{\boldsymbol{d}}\mathbb{Z}^\ell$ on the factor $\mathbb{R}^\ell$. From the results of Section 3, we obtain:

Corollary 9. *With the previous assumptions, there exists a linear stratification* $(\mathcal{S}_m, \mathcal{S}_p)$ *of the projection* $\mathbb{R}^\ell \times \mathbb{R}^p \to \mathbb{R}^p$ *compatible with* $\mathcal{F}$*. This stratification is (w)-regular.* □

Moreover, the construction of Lemmas 6 and 7 shows that the family $\mathcal{F}'$ giving rise to $\mathcal{S}_m$ is stable by the action of $\frac{2\pi}{\boldsymbol{d}}\mathbb{Z}^\ell$, and thus the stratification $\mathcal{S}_m$ is also stable, i.e., for any stratum S_m and any $\boldsymbol{k} \in \mathbb{Z}^\ell$, the set $S_m + 2\boldsymbol{k}\pi/\boldsymbol{d}$ is a stratum. We identify $\mathbb{R}^\ell/(\frac{2\pi}{\boldsymbol{d}}\mathbb{Z}^\ell)$ with $(S^1)^\ell$. Taking the quotient of $\mathcal{S}_m$ by $\frac{2\pi}{\boldsymbol{d}}\mathbb{Z}^\ell$ gives thus rise to a stratification $(\widetilde{\mathcal{S}}_m, \mathcal{S}_p)$ of the projection $(S^1)^\ell \times \mathbb{R}^p \to \mathbb{R}^p$.

Corollary 10. *The stratification* $(\widetilde{\mathcal{S}}_m, \mathcal{S}_p)$ *of the projection* $(S^1)^\ell \times \mathbb{R}^p \to \mathbb{R}^p$ *obtained in this way is (w)-regular.* □

Corollary 11. *Let Z be a subanalytic space and let $g : Z \to \mathbb{R}^p$ be a continuous subanalytic map. Let $(\mathcal{U}, \mathcal{S}'_p)$ be a subanalytic stratification of g such that both $\mathcal{U}$ and $\mathcal{S}'_p$ are (w)-regular and $\mathcal{S}'_p$ is a refinement of $\mathcal{S}_p$. Let $\widetilde{\mathcal{S}}'_m$ be the corresponding refinement of $\widetilde{\mathcal{S}}_m$. Then $(\mathcal{U} \times_{\mathcal{S}'_p} \widetilde{\mathcal{S}}'_m, \mathcal{U})$ is a (w)-regular stratification of the projection $(S^1)^\ell \times Z \to Z$.* □

5. Φ-irregular stratifications and triangulations. Let $X = \Delta_\ell \times E$ be an open subset of $\mathbb{C}^n$ with coordinates $x_1, \dots, x_n$, let D be the divisor $(x_1 \cdots x_\ell)$ and set

$E=\{x_1=\cdots=x_\ell=0\}$. Let $\boldsymbol{d}=(d_1,\dots,d_\ell)$ be a multi-index of positive integers and let

$$\begin{aligned}\rho_{\boldsymbol{d}}:\Delta'_\ell&\longrightarrow\Delta_\ell\\ (x'_1,\dots,x'_\ell)&\longmapsto(x_1'^{d_1},\dots,x_\ell'^{d_\ell})\end{aligned}$$

denote the ramified morphism of multi-degree $\boldsymbol{d}$. We set $X'=\Delta'_\ell\times E$ and $D'=(x'_1\cdots x'_\ell)$.

Let $\varpi:\widetilde{X}\to X$, resp. $\varpi':\widetilde{X}'\to X'$, be the real blowing up of the irreducible components of D, resp. D', and set $\widetilde{E}=\varpi^{-1}(E)\simeq(S^1)^\ell\times E$ resp. $\widetilde{E}'=\varpi'^{-1}(E)\simeq(S^1)^\ell\times E$. We set $\underline{E}'=\mathbb{R}^\ell\times E$ with coordinates $(\theta'_1,\dots,\theta'_\ell,x_{\ell+1},\dots,x_n)$, and we identify $\widetilde{E}$ with $\underline{E}'/\frac{2\pi}{\boldsymbol{d}}\mathbb{Z}^\ell$.

Let $\Phi\subset\mathcal{O}(X')(*D')/\mathcal{O}(X')$ be a good finite family of polar parts of meromorphic functions. By definition, for any two distinct elements φ,ψ of Φ, the divisor of the difference $\varphi-\psi$ is supported on D' and is effective. We can thus write $\varphi-\psi=u(\boldsymbol{x}')\boldsymbol{x}'^{-\boldsymbol{a}}$ for some $\boldsymbol{a}\in\mathbb{N}^\ell$ and u is a unit, i.e., is holomorphic on X' and does not vanish. As a consequence, there is a well-defined real-analytic map

$$\lambda_{\varphi-\psi}=\frac{\varphi-\psi}{|\varphi-\psi|}:X'\smallsetminus D'\longrightarrow S^1,$$

and this map extends to $\widetilde{X}'$. Up to shrinking E, we can assume that there exists a real analytic function

$$\begin{aligned}g_{\varphi-\psi}:E&\longrightarrow\mathbb{R}\\ \boldsymbol{x}=(x_{\ell+1},\dots,x_n)&\longmapsto\operatorname{Arg}u(0,x_{\ell+1},\dots,x_n).\end{aligned}$$

Then $\lambda_{\varphi-\psi}$ lifts to $\underline{E}'$ as

$$\begin{aligned}\Lambda_{\varphi-\psi}:\underline{E}'&\longrightarrow\mathbb{R}\\ (\theta'_1,\dots,\theta'_\ell,x_{\ell+1},\dots,x_n)&\longmapsto L_{\varphi-\psi}(\boldsymbol{\theta}')+g_{\varphi-\psi}(\boldsymbol{x}),\end{aligned}$$

with $L_{\varphi-\psi}(\boldsymbol{\theta}')=\sum_{i=1}^\ell a_i\theta'_i$ $(a_i\in\mathbb{N})$.

Definition 12.

(1) We say that E is small enough with respect to Φ if for any pair $\varphi,\psi\in\Phi$ with $\varphi\neq\psi$, there exists a continuous lift $g_{\varphi-\psi}:\boldsymbol{x}\mapsto\operatorname{Arg}u(0,\boldsymbol{x})$ from $E\to\mathbb{R}$.

(2) Assume E is small enough with respect to Φ. A Φ-irregular stratification of $\widetilde{E}$ is a subanalytic stratification of $\widetilde{E}$ which is compatible with the family of closed subsets induced, after taking the quotient by $\frac{2\pi}{\boldsymbol{d}}\mathbb{Z}^\ell$, by the family of relative hyperplanes $L_{\varphi-\psi}(\boldsymbol{\theta}')+g_{\varphi-\psi}(\boldsymbol{x})=\pm\pi/2+2k\pi$, where k varies in $\mathbb{Z}$ and $\varphi\neq\psi$ in Φ, their intersections and their translates by the lattice $\frac{2\pi}{\boldsymbol{d}}\mathbb{Z}^\ell$.

Corollary 13. *Let $\Phi\subset\mathcal{O}(X')(*D')/\mathcal{O}(X')$ be a good finite family of polar parts of meromorphic functions. Assume that E is small enough with respect to Φ. Then there exists a (w)-regular stratification $(\widetilde{\mathcal{S}},\mathcal{S})$ of $\varpi_{|\widetilde{E}}:\widetilde{E}\to E$ which is Φ-irregular.*

Proof. We denote by p the cardinal of the set of pairs of distinct elements of Φ and we regard the family of functions $g_{\varphi-\psi}$ as defining a real-analytic map $g : E \to \mathbb{R}^p$. We conclude with Corollary 11. $\square$

Corollary 14 **([4, Lem. I.1.3′] and [5, Theorem]).** *Under the same assumptions, there exists a* Φ*-irregular stratification* $\widetilde{\mathcal{S}}$ *of* $\widetilde{E}$ *and a triangulation of* $\varpi_{|\widetilde{E}}$ *which is compatible with* $\mathcal{S}$. $\square$

References

[1] J.P.G. Henry, M. Merle & C. Sabbah – "Sur la condition de Thom stricte pour un morphisme analytique complexe", *Ann. Sci. École Norm. Sup. (4)* **17** (1984), p. 227–268.

[2] V. Navarro Aznar – "Conditions de Whitney et sections planes", *Invent. Math.* **61** (1980), no. 3, p. 199–225.

[3] C. Sabbah – "Morphismes analytiques stratifiés sans éclatement et cycles évanescents", in *Analyse et topologie sur les espaces singuliers (Luminy, 1981)* (B. Teissier & J.-L. Verdier, eds.), Astérisque, vol. 101-102, Société Mathématique de France, Paris, 1983, p. 286–319.

[4] M. Shiota – *Geometry of subanalytic and semialgebraic sets*, Progress in Math., vol. 150, Birkhäuser Boston, Inc., Boston, MA, 1997.

[5] ———, "Thom's conjecture on triangulations of maps", *Topology* **39** (2000), no. 2, p. 383–399.

[6] J.-L. Verdier – "Stratifications de Whitney et théorème de Bertini-Sard", *Invent. Math.* **36** (1976), p. 295–312.

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