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Étude Cohomologique des Variétés Algébriques

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1. Summary

This project, at the crossroads of algebraic geometry and number theory, aims at investigating some aspects relating the arithmetico-geometric and cohomological properties of an algebraic variety.

The notion of cohomology is somewhat the modern culmination of the old idea of 'linearization', which consists in associating to an object with a rich structure (an algebraic variety), families of simpler objects (linear *i.e.* vector spaces or modules with additional data - for instance a cohomology group) supposed to encode lots of the relevant information about the original object but easier to handle and classify.

Abelian cohomology has progressively emerged in topology and complex geometry during the first half of the twentieth century. During this period, first, and second cohomology pointed sets also appear with the study of twists in Galois theory. In the fifties and early sixties, the development of category theory and homological algebra provided the conceptual frame to elaborate systematically a cohomological formalism in abelian categories, paving the way to its diffusion in many areas of mathematics. In particular, it is intrinsically connected to the breathtaking development of modern algebraic geometry, yielding to the introduction of various algebraic Weil cohomologies related by comparison isomorphisms. Around the early seventies, these several cohomological avatars were unified in Grothendieck's fascinating conjectural theory of pure motives, which was later included in the wider theory of mixed motives.

These theories - partly conjectural - naturally give rise to 'conjectures horizon', such as the Hodge and the Tate conjectures, various conjectures on algebraic cycles *etc.* The purpose of ECOVA is to study a

series of problems arising as consequences or particular cases of these 'conjectures horizon', which we will describe in more details in Section 4 and can be roughly classified according to the three following main directions:

(1) Complex and ℓ-adic coefficients: absolute setting (special cases of the Hodge and the Tate conjectures, Mumford-Tate conjecture), variational setting (geometric study of the exceptional loci, representations of the fundamental group, ℓ-independence, Shimura varieties);
(2) Integral, finite, adelic coefficients: absolute setting (obstructions to the integral variant of the Hodge and the Tate conjectures), variational setting (representations of the fundamental group, semi simplicity modulo ℓ, adelic openness and ℓ-independence and Shimura varieties);
(3) Finer questions on algebraic cycles (Higher Abel-Jacobi maps, integral and birational aspects, study of zero-cycles and rational points, mixed motives).

ECOVA gathers young mathematicians working on these questions so that they could share the technics they handle, acquire new ones, initiate and pursue collaborations. As a result, a particular emphasis will be put on the organization of weekly or monthly working groups, series of invited lectures of post-doctoral level, visit to and invitation of foreign specialists, attending conferences. The 48 months of the project will be structured around three conference-like events: an opening meeting where the researchers involved in the project will present and discuss the mathematical problems they are working on, a mid-project meeting, in the spirit of a mixed summer school/research conference, and a closing meeting, with the purpose of presenting the main mathematical achievements of the project in the frame of a wide-audience research conference.

2. Contribution and qualification of the participants

As ECOVA is a project designed for the financial support 'Jeunes Chercheurs', it will only involve a single administrative financial pole - Ecole Polytechnique. So we do not mention it in the table below. Let us mention, however, that the members of the project being scattered over various institutions - Ecole Normale Supérieure, Ecole Polytechnique, Paris 6, Paris 7, Paris 11, Bordeaux, Lyon 1, Strasbourg - we do intend to involve each of these poles (and also I.H.P. and I.H.E.S.), as well as national conference centers (C.I.R.M or I.H.P.) in the organization of the scientifically events schedules all along the project.

Name	Current position	Implication*	Mathematical interests
Benoist Olivier	CR	24 (50%), member	algebraic geometry, moduli spaces of varieties, arithmetic geometry, algebraic cycles.
Cadoret Anna	PA (EP)	38,4 (80%), coord.	Arithmetic geometry, Galois representations, étale fundamental group, étale and Galois cohomology, motives, Shimura varieties, Abelian varieties.
Charles François	PU (Paris 11)	28,8 (60%), member	Arithmetic geometry, Hodge theory, Algebraic cycles, Hyperkähler geometry.
Demarche Cyril	MCU (Paris 6)	24 (50%), member	Arithmetic geometry, Galois cohomology, non-abelian cohomology, algebraic groups and their homogeneous spaces, zero-cycles, local-global principles.
Fu Lie	MCU (Lyon 1)	12 (25%), member	Algebraic geometry, Hodge theory, hyperkähler geometry, algebraic cycles, motives.
Klingler Bruno	PU (Paris 7)	12 (25%), member	Algebraic geometry, Shimura varieties, non-abelian Hodge theory, motives.
Morin Baptiste	CR (Bordeaux)	19,2 (40%), member	Arithmetic geometry, special values of zeta functions, tale and Weil-tale cohomology, motives and motivic cohomology, Deninger's program, (higher) topos theory.
Orgogozo Fabrice	CR, PCC (EP)	38,4 (80%), member	Arithmetic geometry, étale and Galois cohomology, wild ramification, vanishing cycles, (theoretical) computational aspects of étale cohomology.
Pirutka Alena	CR, PCC (EP)	38,4 (80%), member	Arithmetic geometry, Galois cohomology, unramified cohomology, birational invariants, étale and motivic cohomology, Chow groups and integral aspects.
Riou Joël	MCU (Paris 11)	24 (50%), member	Motivic homotopy theory, Algebraic K-theory, étale cohomology.
Wittenberg Olivier	CR (ENS Paris)	31,2 (65%), member	Arithmetic geometry, algebraic geometry, Chow groups, cohomology theories, fundamental groups.

*: In person.month.
EP: Ecole Polytechnique, CR: Chargé de Recherche CNRS, MCU: Maître de Conférences des Universités, PA: Professeur Associé, PU: Professeur des Universités, PCC: Professeur chargé de cours.

3. Evolution of the detailed proposition compared with the pre-proposition

In the next sections, we develop both the scientific and technical features of our project which were only sketched in the preliminary proposition (pré-proposition).

4. Context and positioning of the proposal, scientific description

Before turning to the details, let us make the frame of our project slightly more precise and introduce some notation.

The notion of cohomology is somewhat the culmination of the old idea of 'linearization', which consists in associating to an object with a rich structure (an algebraic variety X), families of simpler objects (linear *i.e.* vector spaces or modules with additional data - for instance a cohomology group $H(X)$) supposed to encode a lot of the relevant information about the original object but easier to handle. For instance, part of the rich structure of X is reflected in properties of Chow groups $CH^i(X)$ – quotients by the rational equivalence relation of the free abelian groups on codimension i integral subvarieties of X. These objects are still complicated (in general, of infinite rank) and difficult to handle but they map to cohomology algebras, which are much nicer. More formally, fix two fields k and K and let $\mathcal{V}_{/k}$ denote the category of smooth projective varieties over k. A (Weil) cohomology theory with coefficients in K associates in a functorial way to every $X \in \mathcal{V}_{/k}$ a finite- dimensional K-algebra $H(X)$ and a cycle class map

$$cl : CH(X) := \bigoplus_i CH^i(X) \to H(X).$$

These data are to satisfy a series of axioms such as the Künneth formula, Poincaré duality, etc. Typical examples are Betti ($H_B(X)$), algebraic de Rham, ℓ-adic ($H_\ell(X)$) and crystalline cohomology theories.

Weil cohomology theories are related by comparison isomorphisms. For instance, if $k \subset \mathbb{C}$, one has a functorial isomorphism $H_B(X) \otimes \mathbb{Q}_\ell \tilde{\to} H_\ell(X)$. This, together with the fact that the enriched target categories are Tannakian, is at the origin of the theory of motives envisioned by Grothendieck. More precisely, fix a Weil cohomology H. The category of pure H-motives is a rigid pseudo-abelian tensor category $\mathcal{M}^H$ built on H-correspondences, together with a functor of tensor categories $\mathfrak{h} : \mathcal{V}_{/k} \to \mathcal{M}^H$ According to the so-called standard conjectures, $\mathcal{M}^H$ is Tannakian, hence is the category of finite-dimensional representations of a pro-algebraic group $\mathcal{G}^H$ over K. The comparison isomorphisms ensure that $\mathcal{G}^H$ is essentially independent of H. While this (in particular the existence of $\mathcal{G}^H$) is conjectural, André showed that it is possible to construct an unconditional version of $\mathcal{G}^H$ in characteristic zero by considering motivated correspondences.
Let $\mathcal{G}_X^B$, $\mathcal{G}_X^\ell$ and $\mathcal{G}_X^{mot,B}$ denote the Galois group of the tensor subcategory generated respectively by $H_B(X)$ in the category of $\mathbb{Q}$-Hodge structures, $H_\ell(X)$ in the category of continuous finite-dimensional $\mathbb{Q}_\ell$-representations of Γ_k and the Betti motive of X in the category of motivated Betti motives. The Hodge (resp. the Tate) conjecture for the powers of X implies that the tensor subcategory generated by the Betti (resp. the ℓ-adic) motive of X is Tannakian and that its Galois group coincides with $\mathcal{G}_X^B$ (resp. $\mathcal{G}_X^\ell$). In particular, $\mathcal{G}_X^B$ should coincide with $\mathcal{G}_X^{mot,B}$ and $\mathcal{G}_X^\ell$ should coincide with $\mathcal{G}_X^{mot,B} \otimes \mathbb{Q}_\ell \simeq \mathcal{G}_X^B \otimes \mathbb{Q}_\ell$. As a by-product of these conjectural isomorphisms, the groups $\mathcal{G}_X^\ell$ should be independent of ℓ and reductive.

This general Tannakian picture is the motivating background of the questions we address in the first part of our project ((1) complex and ℓ-adic coefficients), which is split as follows in the following more detailed scientific description:

(1) Complex and ℓ-adic coefficients:

- Absolute setting (special cases of the Hodge and the Tate conjectures - 4.1.1);
- Variational setting (geometric study of the exceptional loci, representations of the fundamental group, ℓ-independence, Shimura varieties - 4.1.2).

The second ((2) Integral, finite, adelic coefficients) and third ((3) Finer questions on algebraic cycles) parts of our project are split as follows:

(2) Integral, finite, adelic coefficients:

- Absolute setting (obstructions to the Hodge and the Tate conjectures - 4.2.2.1);
- Variational setting (representations of the fundamental group, semi simplicity modulo ℓ, adelic openness, ℓ-independence and Shimura varieties - 4.2.1);

(3) Finer questions on algebraic cycles:

- Higher Abel-Jacobi maps - 4.2.2.2 (in the variational setting, see 4.1.2.2, Example 2);
- Integral and birational aspects - 4.2.2.4;
- Study of zero-cycles - 4.2.2.3, and rational points;
- Mixed motives - 4.2.2.5.

Along these general guidelines, we recall the major open conjectures and formulate a sample of more specific questions we want to consider in the frame of this project. For the later, we explain what are the difficulties to overcome and sketch strategies along which we plan to tackle them.

4.1. **Part 1: Complex and ℓ-adic coefficients - the philosophy of pure motives.**

4.1.1. *The conjectures of Hodge and Tate.*

4.1.1.1. *Overview.* Let H be one of the Weil cohomology theories considered above. The target category of H comes with additional structures. These 'remember' algebro-geometric data which are crucial in the process of recovering information about X. If $k \subset \mathbb{C}$, the Betti cohomology group $H_B(X) := H(X_{\mathbb{C}}^{an}, \mathbb{Q})$ considered as a $\mathbb{Q}$-vector space encodes properties of the underlying topological space of $X_{\mathbb{C}}^{an}$. The complex structure on $X_{\mathbb{C}}^{an}$ is reflected through the existence of a so-called Hodge structure on $H_B(X)$. Similarly ℓ-adic cohomology $H_\ell(X) := H(X_{\overline{k}}, \mathbb{Q}_\ell)$ is endowed with a continuous action of the absolute Galois group Γ_k of k, which contains information about X as an arithmetic variety defined over k. The Hodge and Tate conjectures assert that the realization functor from the category of motives with respect to homological equivalence to these enriched categories is fully faithful. This statement can be phrased in a more concrete way as follows.

If $k = \mathbb{C}$, the image of the cycle class map for Betti cohomology $CH^i(X) \otimes \mathbb{Q} \stackrel{cl^i}{\to} H_B^{2i}(X)$ is contained in the subspace $Hdg^{2i}(X, \mathbb{Q})$ of so-called Hodge classes and the Hodge conjecture predicts that it should coincide with it. The Hodge conjecture is known for divisors ('Lefschetz $(1,1)$-theorem') and in a few other cases, mostly obtained from the case of divisors. It remains widely open in general.

If k is finitely generated over its prime subfield, the image of the cycle class map for ℓ-adic cohomology $CH^i(X) \otimes \mathbb{Q}_\ell \stackrel{cl^i}{\to} H_\ell^{2i}(X)(i)$ is contained in the group $H_\ell^{2i}(X)(i)^{\Gamma_k}$ of Galois-invariant classes and the Tate conjecture predicts that it should coincide with it. The Tate conjecture is known for divisors for divisors on products of curves and abelian varieties (Tate, Faltings) and in a few other cases. As the Hodge conjecture, it remains widely open in general; even the case of divisors on surfaces is open. Except for finite fields, the ℓ-independence of the Tate conjecture is not known.

4.1.1.2. *Perspectives*. Recently, the Tate conjecture for $K3$ surfaces has been established in full generality by Maulik, Charles, Madapusi Pera. A first approach, pioneered by Maulik and completed by Charles, relied heavily on complex analytic arguments due to Borcherds, while Madapusi Pera's proof builds on work of Kisin on integral models of Shimura varieties. A more recent – and more direct – proof by Charles (in preparation) relies on methods from hyperkähler geometry and moduli spaces of sheaves. As in the proof by Charles-Markman of the standard conjectures for certain holomorphic symplectic varieties – a special case of the Hodge conjecture – this emphasizes the role of a detailed understanding of the fine geometric structure of moduli spaces of stable sheaves and of complex-analytic, non projective techniques in the study of motivic and arithmetic problems. We hope that a better understanding of purely analytic methods might shed light on more general cases of these conjectures. In more general settings, analytic aspects, both at the finite and the infinite places, should play a role. This is apparent of course in the proof by Faltings of the Tate conjecture for endomorphisms on abelian varieties which uses heights on moduli spaces. It seems conceivable that p-adic and complex analysis, in the spirit of Arakelov geometry, might shed some light to these conjectures, see for instance work of Bost and Chambert-Loir on algebraization theorems or K. Kato's work on heights of motives. Arakelov properties of special subvarieties of Shimura varieties is a related and active subject. The relevance of the algebraic dynamics of the Hecke correspondences has appeared in work of Charles on Frobenius distribution for pairs of elliptic curves over number fields [Ch15].
We can mention two other, perhaps more concrete problems. The first would be to construct curves on supersingular surfaces of high degree in $\mathbb{P}^3$ over finite fields using methods coming from the proof of the Tate conjecture for $K3$ surfaces. The second one would be to investigate how the framework we just described can be applied to the related but different study of periods of algebraic varieties. This study has been started in work of Bost-Charles [BoCh15].

4.1.1.3. *ℓ-independence*. As a by-product of the standard and the Tate conjectures, the groups $\mathcal{G}_X^\ell$ should be independent of ℓ, reductive and coincide with $\mathcal{G}_X^B \otimes /Q_\ell$. This leads to the investigation of ℓ-independence properties in so-called systems of geometric ℓ-adic Galois representations ('geometric systems' for short), a problem which is also related, *via* the Langlands' philosophy, automorphic representations.. The notion of geometric systems encodes specific features arising from etale cohomology: (R) ramified at finitely many places (Grothendieck), (dR) de Rham (Fontaine, Messing, Faltings *etc*), (W) satisfying the Weil conjectures (Deligne). The Fontaine-Mazur conjecture predicts that a system of *irreducible* geometric ℓ-adic Galois representations indeed arises from geometry and, in particular, the Zariski closure of the Galois images (which are reductive by the irreducibility assumption) should be ℓ-independent. Important results (on the reductive and semi-simple ranks, group of connected components, root system *etc*) in this direction were obtained by Serre, Larsen-Pink, Hui using among others Frobenius tori, the machinery of reductive and ℓ-adic Lie groups or local algebraicity properties of abelian ℓ-adic representations related to class-field theory. The Fontaine-Mazur conjecture provides a conjectural characterization of the irreducible constituents of the subcategory generated by geometric ℓ-adic Galois representations but this leaves the issue of semi simplicity aside. So, the problem can be split into two parts: ℓ-independence in systems of semi simple geometric ℓ-adic Galois representations and Irreducibility/semi simplicity in systems arising from geometry.

As the notion of geometric system only encodes informations about the semi simplification of the representations, one cannot expect to obtain results about semi simplicity only under the defining conditions (R), (dR), (W). To get new insights, one should exploit refined informations coming from geometry or technics from automorphic representations. This already appears in Serre's pioneering work on Frobenius tori, where he uses information about the valuations of the eigenvalues of Frobenii. One first track would be to exploit further this circle of ideas, following the way paved, among others, by Pink and Chin (in combination with the geometric Langlands' correspondance for GL_2 proved by L. Lafforgue). Another track to investigate are induction arguments using Lefschetz pencils in the spirit of Deligne's proof of the Weil conjectures.

A different way to attack these questions is to formulate them in a variational setting. Indeed, systems of geometric ℓ-adic Galois representations can be deformed in family, a situation which is often more tractable due to the fact, precisely, that additional information arises from the whole family.

4.1.2. *Variational approach, representations of étale fundamental group.* More precisely, Weil cohomologies can be sheafified that is, given a smooth projective morphism $f : X \to B$, the cohomology of the fibers $H(X_b)$ are the stalks $\mathcal{H}(X)_b$ of a sheaf $\mathcal{H}(X)$ on B. One main problem is to understand the arithmetico-geometric properties of the various possible special loci (denoted generically by B^{ex} in the following), that is, the locus of points b where $H(X_b)$ behaves differently than at the generic fiber X_η.

4.1.2.1. *Complex setting - the Hodge locus.* For instance, in the setting of Betti cohomology, one can consider the Hodge locus B^{ex} which parametrizes the points b of B whose fibers $\mathcal{H}(X)_b$ have an exceptional Mumford-Tate group. It is known that the corresponding motivic locus $B^{mot,ex}$ which parametrizes the points b of B where the motivated motivic Galois group degenerates is a countable union of sub-varieties of B distinct from B. As the Hodge conjecture predicts that B^{ex} should coincide with $B^{mot,ex}$, B^{ex} as well should be a countable union of irreducible algebraic subvarieties of B. This was shown by Cattani-Deligne-Kaplan, in 1995. A generalization of this locus has been recently studied by Brosnan-Pearlstein, Saito and Schnell, who show, in particular that the zero-locus of an admissible normal function on B is algebraic. These results serve as a guideline to study what occurs in the ℓ-adic setting, where much less is known but where, if one believes in the Hodge and Tate conjectures, similar results should hold.

4.1.2.2. *ℓ-adic setting.* The smooth-proper base change theorem reduces the problem to studying the ℓ-adic representations of the étale fundamental group $\pi_1(B)$ on $H_\ell(X_\eta)$ (then, for every $b \in B$, the pullback of this representation *via* $\Gamma_{k(b)} \xrightarrow{b} \pi_1(B)$ identifies with the natural representation of $\Gamma_{k(b)}$ on $H_\ell(X_b)$). In that case, one gets an exceptional locus B^{ex}_ℓ for every ℓ and the questions raised by the Hodge and the Tate conjectures can be split in two categories:

(a) ℓ-independence;
(b) Arithmetico-geometric nature of the B^{ex}_ℓ.

A weak variant of (a) would be to show (a') $\cap_\ell B^{ex}_\ell \neq \emptyset$ (or $\cup_\ell B^{ex}_\ell \neq X$).

The expected answers to problems (a) and (b) depend on what we mean by 'behaves differently than at the generic fiber'.

Example 1: One can consider the locus B^{ex}_ℓ which parametrizes points $b \in B$ where (the Zariski closure of) the image of $\Gamma_{k(b)}$ is not open in (the Zariski closure of) the image of $\pi_1(B)$. This is the analogue of the Hodge locus in Betti cohomology (Subsection 4.1.2.1) and, according to the Tate conjecture, it should coincide with $B^{mot,ex}$ hence be independent of ℓ and a countable union of irreducible algebraic subvarieties. It is known to be independent of ℓ in the case of abelian varieties or when the Zariski closure of the image of $\pi_1(X_{\overline{k}})$ has only simple factors of type A_n. It is also known that B^{ex}_ℓ contains $B^{mot,ex}$ [C12] and that $B^{ex}_\ell \cap B(k)$ is thin. When B is a curve and k is finitely generated, one also knows that B^{ex}_ℓ contains only finitely many points of bounded degree provided the image of the geometric étale fundamental group $\pi_1(B_{\overline{k}})$ is Lie-perfect (which holds for ℓ-adic cohomology - Deligne) (Cadoret-Tamagawa [CT12b], [CT13a]). But, for instance, one knows that the set of closed points in B^{ex}_ℓ is not of bounded height as shown by the 'baby-case' of modular curves where B^{ex}_ℓ is the CM-locus. More generally, the working examples for such questions are Shimura varieties. In this setting, the formulation of the problems are more explicit. For instance, in view of the Mumford-Tate conjecture, B^{ex}_ℓ should be the union of all special subvarieties (other than the ambient Shimura variety). Also, the André-Oort conjecture give a more precise conjectural description of the arithmetics-geometric structure of B^{ex}_ℓ in terms of special points. These conjectural statements still make sense in the general setting of ℓ-adic representation arising from geometry and we intend to explore systematically the possibility of extending results from the theory of Shimura varieties to representations of étale fundamental groups arising from geometry. Conversely, we hope that general results for representations of étale fundamental groups arising from geometry will give new insights in the specific setting of Shimura varieties. Here is a sample of questions illustrating more concretely our purposes:

(Shimura to general): If B is a curve and the Zariski closure of the image of $\pi_1(B)$ satisfies some reasonable assumptions (*e.g.* contains no factor of type A_1), can we show there are only finitely many $b \in B$ such that the connected component of the Zariski closure of the image of $\Gamma_{k(b)}$ is abelian? This is an analogue of the André-Oort conjecture for curves. A rough strategy would be to prove that degeneration to abelian image produces too many rational points on some étale covers of B, contradicting the finiteness results of [CT12b], [CT13a]. This problem is related to the question of constructing Hecke correspondances in the general setting. In some sense, these exist group-theoretically (using the formalism of Galois category) but what is unclear is whether they correspond indeed to algebraic correspondances.

(Shimura): Try and attack the Mumford-Tate conjecture using integral models of Shimura varieties that is, starting with a g-dimensional abelian variety over a number field with given ℓ-adic monodromy strictly smaller than GSp_{2g}, show that it lies on a special sub variety of the Siegel moduli variety by showing that its reduction modulo p lies on the reduction modulo p of the same special sub variety for 'lots of' p. This strategy could also be settled in some special cases of more general situation; it suggests that integral structures should be exploited.

(General to Shimura): Formulating and proving what the analogue of the above mentioned fact that B_ℓ^{ex} contains only finitely many points of bounded degree when B is a curve should be in the particular higher-dimensional setting of Shimura varieties should shed light on the 'geometrico-arithmetic' phenomena hidden behind it and hopefully yield interesting results on the arithmetical properties of Shimura varieties. A too optimistic formulation (but illustrating what kind of statement one may expect) is that for every integer $d \geq 1$ there are only finitely many special sub varieties defined over a number field of degree $\leq d$.

In the general setting, we would like to develop the variational strategy of [C15a], where it is shown that for $b \notin \cap_\ell B_\ell^{ex}$, the unipotent radical of the Zariski-closures of the image of $\Gamma_{k(b)}$ and $\pi_1(B)$ coincide. If for every ℓ one knows that the Zariski-closure of the image of $\pi_1(B)$ is reductive (for instance in the generic case, where it is symplectic or orthogonal), this implies that for every $b \notin \cap_\ell B_\ell^{ex}$ the corresponding geometric family of ℓ-adic representation will satisfy the semi-simplicity conjecture (for every prime ℓ). In [C15a], one also reduces the problem of the ℓ-independence of B_ℓ^{ex} to showing that the center of the Zariski-closures of the image of $\Gamma_{k(b)}$ and $\pi_1(B)$ coincide for $b \notin \cap_\ell B_\ell^{ex}$ and every ℓ. The same statement holds with the connected component of the center so, the key problem to study seems to be the ℓ-independence of the group of connected components of the center.
One may also expect that the smallness of B_ℓ^{ex} is related to analytic properties. Tracks to explore this include p-adic technics used by Maulik-Poonen [MauP10] or Kim [K05] and Hadian-Jazi [H11].

Example 2: The result of [CT12b], [CT13a] also applies to the specialization of the first higher-dimensional Abel-Jacobi maps attached to a smooth projective scheme $X \to B$ (and, in particular, to 1-motives). Here, the representations at stake are of slightly different nature and these results are related to the study of the filtrations introduced on Chow groups (Subsection 4.2.2.2). For instance, in this setting, the ℓ-independence of B_ℓ^{ex} would follow from the Bloch-Beilinson-Jannsen conjecture about the injectivity of the first higher-dimensional Abel-Jacobi maps over number fields. Also, when $i = 1$ and $X \to B$ is an abelian scheme, it was proved by Silverman that B_ℓ^{ex} is of bounded height and independent of ℓ, by showing that the height pairing at the generic fiber is the limit of the height pairings at the closed fibers. It is natural to try and extend this argument by using the height pairings constructed by Beilinson, Bloch, Gillet-Soulé on higher Chow groups. If one restricts to the specialization of one single algebraic cocycle, B_ℓ^{ex} coincides with the vanishing locus of the corresponding ℓ-adic normal function. In that case and in view of the above mentioned result of Brosnan-Pearlstein, Saito and Schnell, it is expected that B_ℓ^{ex} is algebraic (hence finite if B is a curve). This questions was raised in Charles [Ch10]. It is not clear whether the specialization results of Cadoret-Tamagawa extends to higher-rank Galois cohomology groups but this is also be a problem to investigate.

Inverse problems: It would be nice to have at disposal techniques to construct families of smooth proper schemes with a given geometric monodromy. The construction of universal abelian schemes over Shimura varieties provide examples limited to abelian schemes but what would really be interesting is to construct examples which do not reduce (say by a Tannakian construction) to the cohomology of abelian varieties, where things are usually easier. Such technics may include those developed by Katz, nearby and vanishing cycles, families with non-trivial automorphism group, cohomology of Shimura varieties *etc.*

Weil cohomology theories have remarkable properties but forget non-trivial information encoded in more intricate invariants such as the underlying integral cohomologies (which may contain torsion) and intersection rings, especially the Chow ring (which may be of infinite rank). Here is a sample of questions related to these issues which are somewhat 'beyond' the philosophy of pure motives. For

4.2. **Part 2: Integral coefficients and Finer information on algebraic cycles - Beyond the philosophy of pure motives.**

4.2.1. *Variational setting.*
4.2.1.1. *Adelic and finite coefficients.* The philosophy of pure motives predicts what the algebraic groups $\mathcal{G}_X^\ell$ should be. But number theorists are also interested in determining precisely the image of Γ_k inside $\mathcal{G}_X^\ell(\mathbb{Z}_\ell) := \mathcal{G}_X^\ell(\mathbb{Q}_\ell) \cap \mathrm{GL}(H_\ell(X_{\overline{k}}, \mathbb{Z}_\ell))$ or $\prod_\ell \mathcal{G}_X^\ell(\mathbb{Z}_\ell)$. In the absolute setting, these questions were partially investigated for representations attached to elliptic curves and Abelian varieties (Serre) or for system of ℓ-adic geometric representations (Larsen). But, here again, the variational approach seems to be particularly appropriate. For instance, using it, Bilu-Parent recently proved one of the two remaining cases of Serre's uniformity conjecture for the adelic image of representations attached to elliptic curves. In [C15b], a strategy is elaborated to prove adelic openness results with $\mathcal{G}_{X_b}^\ell(\mathbb{Z}_\ell)$ replaced by the image of $\pi_1(B)$. This strategy is carried out in the case of abelian varieties (and 1-motives), where it is shown that for $b \notin B^{ex}$, the (adelic) image of $\Gamma_{k(b)}$ is open in the (adelic) image of $\pi_1(B)$. The most difficult step is to show that the reductions modulo ℓ of the images of $\Gamma_{k(b)}$ and $\pi_1(B)$ coincide for $\ell \gg 0$. There, one uses as a crucial ingredient the variant modulo ℓ of the Tate conjectures for abelian varieties (Faltings). This raises at least two interesting questions:

(a) Compute explicitly the (adelic) image of $\pi_1(B)$ when B is a Shimura variety. To carry this out, the point is to make the reciprocity map describing the action of Galois on the connected components of the canonical model of the Shimura variety explicit.

(b) Find substitutes for the variant modulo ℓ of the Tate conjectures for abelian varieties in the more general case of étale cohomology with $\mathbb{F}_\ell$-coefficients. As observed by Cadoret (in preparation), the semi simplicity modulo ℓ follows from the ℓ-adic Tate conjectures but the case of the fullness modulo ℓ is more difficult. Using Serre's tame inertia conjecture, Hui [Hu14] obtained partial results in this direction.

Let us point out that sparsity results in the spirit of [CT12b], [CT13a] for $\mathbb{F}_\ell$-coefficients (and $\ell \gg 0$) have been obtained recently [ElHK12], [CT13b], [CT14a], providing evidences for the generalization of the results of [C15b] to arbitrary adelic representations arising from geometry.
Base field of positive characteristic. When the base field is of positive characteristic, the above questions still make sense and, in the absolute case, they are sometimes easier to deal with. But in the

variational setting, some difficulties arise, due in particular to the fact that we no longer have at disposal the comparison between Betti and ℓ -adic cohomologies, which makes the control of the image of $\pi_1(X_{\overline{k}})$ more difficult. As an example, in characteristic 0, the fact that $\pi_1(X_{\overline{k}})$ acts semi-simply on étale cohomology with coefficients in $\mathbb{F}_\ell$ for $\ell \gg 0$ can be deduced from the analogue result for Betti cohomology with coefficient in $\mathbb{Q}$ (Deligne) by a modulo-ℓ argument [CT11] based on a theorem of Nori and which really exploits the underlying integral data provided by Betti cohomology with coefficient in $\mathbb{Z}$. This no longer works in positive characteristic, where the question of semi simplicity is still open. We hope to be able to handle this issue by resorting to the Langlands' conjecture for GL_n [L02] which should ensure, somewhat, that the image of $\pi_1(X_{\overline{k}})$ modulo ℓ is as large as possible for $\ell \gg 0$. i Another track to explore might be to lsearch for an integral structure on ℓ-adic cohomology in positive characteristic, possibly by comparison with other kind of cohomologies.

4.2.1.2. *Uniform constructibility, ramification and complexity of ℓ-adic sheaves*. After P. Deligne proved the Riemann hypothesis over finite fields and applied it to the study of exponential sums (SGA $4\frac{1}{2}$, [Sommes trig.]), N. Katz and G. Laumon proved some kind of Riemann-Roch theorem for the ℓ-adic Fourier transform and used it to get uniform upper-bounds on exponential sums ([KL85]). One of the main ingredient of their proof is that some cohomological operations preserve "uniform constructibility" of families of ℓ-adic sheaves. (Here "uniform constructibility" means, roughly speaking: existence of a common stratification over which the sheaves are locally constant and common upper-bounds on the ranks of the fibers.). Their result is a *generic* over $\mathbb{Z}$: it uses Hironaka's theorem over $\mathbb{Q}$. This result has recently been used by J.-P. Serre to prove that if k is a number field, X a scheme separated of finite type over k and i an integer, there is a finite extension k' of k such that the diagonal image of the absolute Galois group of k' in $\prod_\ell \mathrm{GL}(H^i(X_{\overline{k}}, \mathbb{Q}_\ell))$ coincide with the product of the images. A similar question makes sense in equal characteristic $p > 0$ and has been studied (circa 2012-2013) by two groups of authors (G. Böckle, W. Gajda, S. Petersen; A. Cadoret, A. Tamagawa). In 2010, F. Orgogozo proved an analogue of Katz-Laumon's theorem without assuming that the generic characteristic is zero; this could be used to prove one of the key result of [BGP13]. The proof given in [O13] uses de Jong's resolution of singularities and the result applies to uniformly constructible *tame* families of sheaves. A most natural question is to try to consider wildly ramified sheaves. There is a good hope that one could improve the results of *op. cit.* to obtain more general statements (but less precise upper-bounds) than those in [FKM13] (motivated by questions on exponential sums and Zhang's recent theorem on primes), or [D12] (motivated by motivic/algebraicity results). In *op. cit.* the "complexity" of sheaves, over a curve, is measured by ranks and Swan conductors. In general, one would probably have to rely on A. Abbes and T. Saitô's work on wild ramification.

4.2.1.3. *Vanishing cycles*. Let S be a Noetherian scheme, $f : X \to S$ be a morphism of finite type and ℓ a prime number invertible on S. For each geometric point x of X, denote by $f_x : X_{(x)} \to S_{(f(x))}$ the morphism induced on the strict localizations ("small balls"). In general, the complexes $Rf_{x\star}\mathbb{Q}_\ell$ are not constructible and their formation doesn't commute with base change. It has been proved by C. Sabbah (characteristic zero, 1981) and F. Orgogozo (any characteristic, 2006) that, after a suitable blowing up of S, the morphisms f_x behave like proper morphisms. This is a basic result to establish a good theory of vanishing cycles over bases of dimension higher than 1. This theory has recently been used by L. Illusie (2013-2014) to establish a general Thom-Sebastiani theorem; according to V. Lafforgue (*private communication*), Orgogozo's result could be also used in his joint work with A. Genestier on Langlands correspondance. However, in positive characteristic, the theory is not as well understood and many natural questions —some of them asked by V. Lafforgue— are still to be settled. One would like for example to have results analogous to those of [Mai13] (over $\mathbb{C}$), where *iterated* vanishing cycles functors are considered and their possible good behaviour is related to a *characteristic variety* of the coefficient sheaf. One could hope to prove similar results in positive characteristic; the following gives a strong impression that interesting new results are within reach. Let $f : X \to Y$ be a morphism of smooth schemes over an algebraically closed field, $\mathcal{F}$ a smooth $\mathbb{Q}_\ell$ sheaf over the complement $U = X - D$ of a simple normal crossings divisor. Answering an old question of P. Deligne,

T. Saitô defined (under some further technical assumptions) the characteristic cycle $\mathrm{Char}(\mathcal{F})$ of $\mathcal{F}$ (in the cotangent bundle) and proved in [Sa13] that if f is non-characteristic with respect to $\mathrm{Char}(\mathcal{F})$ (a simple geometric condition) then the pair $(f, j_!\mathcal{F})$ is *universally acyclic.* (Here j is $U \to X$ and being universally acyclic essentially means that the $Rf_{x_\star}(j_!\mathcal{F})$ have no cohomology in degree > 0.)

4.2.2. *Absolute setting.*

4.2.2.1. *Integral coefficients.* One main question is to understand the cokernels of the integral cycle maps $CH^i(X) \to Hdg^{2i}(X, \mathbb{Z})$ if $k = \mathbb{C}$ or $CH^i(X) \otimes \mathbb{Z}_\ell \to H^{2i}(X_{\overline{k}}, \mathbb{Z}_\ell(i))^{\Gamma_k}$ if k is finitely generated. There are examples where these cokernels are known to be nontrivial, but still very few cases are understood. This yields a question to describe the structure of non-algebraic classes and to understand the obstructions to the validity of these integral versions of the Hodge and Tate conjectures. Clarifying how the integral versions may fail is also important in attempt to prove these conjectures (with coefficients in $\mathbb{Q}$ or $\mathbb{Q}_\ell$). Over $\mathbb{C}$, using a topological argument, Atiyah and Hirzebruch gave examples of non-algebraic Hodge classes, which have been also explained in termes of complex cobordism in a recent work of Totaro. Their construction can be also adapted to provide counterexamples to the integral Tate conjecture, with torsion [CTSz10] or non-torsion [PiYa14] non-algebraic classes. All these examples use classifying spaces of finite or exceptional algebraic groups. From more geometric point of view, Kollár managed to give counterexamples to the integral Hodge conjecture, as some hypersurfaces of divisible degree in $\mathbb{P}^4$; Griffiths and Harris conjectured the same phenomenon in any degree $d \geq 6$. By a refinement of Kollár's method, Hassett and Tschinkel gave counterexamples for the integral Tate conjecture over $\bar{\mathbb{Q}}$ or over a separable closure of $\mathbb{F}_p(t)$. For the integral Tate conjecture over $\bar{\mathbb{F}}_p$, one has no similar geometric examples. The obstructions for a cohomology class to be algebraic include cohomological arguments of motivic nature (as for example motivic Steenrod operations in a work of Pirutka-Yagita [PiYa14]) and some birational invariants coming from Galois cohomology of the function field of X (for instance, the unramified cohomology group in degree 3 for codimension 2 cycles [CTV12, CTK12]). These last groups are intensively studied. In a work of Pirutka [Pi11], this leads to a counterexample to a stronger version of the integral Tate conjecture. There are also some cases when the integral versions do hold. In particular, the integral Hodge conjecture have been established by C. Voisin for uniruled or Calabi-Yau threefolds [Vo06], as well as for cubic fourfolds [Vo13], this argument from complex geometry has been extended in the context of the integral Tate conjecture, together with the method of normal functions of Zucker, in a work of Charles and Pirutka [ChPi13], who established the integral Tate conjecture for cubic fourfolds over finite fields. Understanding the obstructions and the cases when the integral versions hold is one of directions of our project; the new tools to attack these problems, such as motivic methods or birational invariants such as unramified cohomology groups, are beeing developed, as we explain above.

4.2.2.2. *Kernel of the cycle class map.* The kernel of the cycle class map has a very rich structure. While results of Mumford, Griffiths et al. show that these groups can be very large, various invariants, conjectural or not, allow a cohomological study of theses cycles, though they are homologically trivial. Techniques involve higher Abel-Jacobi maps in Hodge theory or étale cohomology, study of Bloch-Beilinson filtrations, etc.

4.2.2.3. *Arithmetical study: zero-cycles, rational points and structural questions.* When k is an 'arithmetic' field, understanding the set $X(k)$ of k-rational points of $X \in \mathcal{V}_{/k}$ and in particular determining whether $X(k)$ is nonempty, is a central question. One can wonder if the existence of a zero-cycle of degree one implies the existence of a rational point (as e.g. for quadrics by the classical Springer's theorem), which is of special interest for homogeneous spaces of algebraic groups, and is related to Galois cohomology of algebraic groups. Another problem of our investigation is the study of local-global principles and obstructions to these principles over global fields. In particular, a conjecture of Colliot-Thélène, Kato and S. Saito (see [KS86], §7 and [CT95], conjecture 1.5) states that on any smooth proper variety over a number field, the existence of a zero-cycle of degree one should be equivalent to the existence of a collection of local zero-cycles of degree one whose cycle classes come from a single global étale cohomology class. A special case of great interest is the case of homogeneous spaces with finite stabilizers, which is related to Galois cohomology of finite groups and to inverse Galois problems (see for instance [De10] for partial results about rational points). There are two recent developments concerning this conjecture. First, the case of homogeneous spaces with connected stabilizers was solved by Liang ([Lia13]). Previously, even the case of toric varieties was open and seemed difficult; thus one might hope to make progress in the case of abelian varieties as well. Secondly, a recent joint work of Y. Harpaz and O. Wittenberg settles the case of fibrations into rationally connected varieties over the projective line when the fibers satisfy the conjecture. It would be very desirable to extend these results to more general bases and more general fibers (*e.g.*, families of elliptic curves).

The structural and finiteness properties of the Chow group are also sensitive to the base field. If one is interested in the birational properties of $X \in \mathcal{V}_{/k}$ - that is those encoded in the field of functions $k(X)$ of X - for instance whether X is rational (resp. unirational, resp. stably rational), *i.e.* whether $k(X)$ is a purely transcendental extension of k (resp. a subfield of such an extension, resp. becomes purely transcendental after adding some independent variables) these universal properties could be very useful. For instance, in the series of recent works [Vo13a], [CTPi14], [Be14a],[Be14b], [T15], [HKT15], one shows that a very general smooth complex variety X is not stably rational if the group $CH_0(X)$ is not universally trivial (i.e. isomorphic a $\mathbb{Z}$) for the following families of varieties : quartic double solids (Voisin), quartic threefolds (Colliot-Thélène and Pirutka), double covers of $\mathbb{P}^3$ branched along a sextic (Beauville), double covers of $\mathbb{P}^4$ or $\mathbb{P}^5$ branched along a quartic (Beauville), hypersurfaces in $\mathbb{P}^{n+1}$ of degree $\geq 2\lceil \frac{n+2}{3} \rceil$ (Totaro), some conic bundles (Hassett-Kresch-Tschinkel). We think that this approach could be applied for other families of varieties and we would be in particular interested in the cases of cubic hypersurfaces, or for problems related to the deformation properties of the rationality.

For $X \in \mathcal{V}_{/k}$ where k is a p-adic field, a wide domain of investigation is the study of the properties of cycles on X which could be understood *via* the geometry of the special fibre of a proper model of X over the ring of integers of k. Here one can mention a recent important progress of K. Sato and S. Saito, who establish that $CH_0(X)/\ell$ is finite for $\ell \neq p$ by using the group of cycles for varieties over a ring and the properties of the cycle class map for 1-cycles on a proper model of X to the convenient cohomology group.

4.2.2.4. *Birational questions and Galois cohomology.* ℓ-adic cohomology is not a birational invariant of varieties in $\mathcal{V}_{/k}$. For the birational properties of $X \in \mathcal{V}_{/k}$ as above one can consider the Galois cohomology groups of $k(X)$ and their subgroups of unramified cohomology. In general, the latter are difficult to compute, but in some cases they allow to provide examples of unirational and non rational varieties. The degree 3 unramified cohomology groups have been also related to the failure of the integral versions of the Hodge and Tate conjecture for codimension two cycles (recent works [CTV12, CTK12]). The computation of unramified cohomology groups of specific families of algebraic varieties over special fields, be it finite fields, local fields, global fields or separably closed fields is therefore a rich field of study and provides a part of concrete problems in the frame of our project. In particular, in the case of homogeneous spaces of algebraic groups, on the one hand the degree 2 unramified cohomology groups were computed in a quite general setting in the recent work [BDH13] by Borovoi, Demarche and Harari, but on the other hand, the computation of the degree 3 unramified cohomology group over a separably closed field is already an open question (only particular cases are known, due to Saltman [S95]). Another example of special interest is the case of threefolds fibred over a curve over a finite field, where understanding of these groups is also related to a local-global principle for zero-cycles on the generic fibre.

4.2.2.5. *Mixed motives and motivic homotopy.* Methods inspired by algebraic topology, such as cobordism and Steenrod operations have proved useful in the study of algebraic cycles, e.g. in Totaro's work on counterexamples to the integral Hodge conjecture [T97].
More systematic approaches have been developped by Voevodsky and others. This implies that one has

to consider not only the cohomology of smooth and projective varieties but also of "open" varieties and sometimes of singular varieties. This is interesting even if one is primarily interested in the cohomology of smooth and projective varieties as it was shown for instance by Wildeshaus in the study of the cohomology of Shimura varieties [W12].

The framework of Voevodsky's category of mixed motives and its homotopical refinement (motivic homotopy theory, or $\mathbf{A}^1$-homotopy) involves "higher" invariants. For instance, motivic cohomology is a bigraded cohomology theory that generalises Chow groups in the same way as "higher" algebraic K-theory generalises the usual Grothendieck group of vector bundles (K_0). When we represent motivic cohomology groups on a plane with the gradings as coordinates, Chow groups appears as a stripe of slope $\frac{1}{2}$. Whereas the statement of the Bloch-Kato conjecture only involves basic or concrete invariants of fields, the celebrated proof by Rost and Voevodsky [R14] needs many motivic cohomology groups that are really "higher" in the sense that they are not on this Chow groups stripe. It is even more general as one has to use that motivic invariants are not only associated to varieties but also to "motivic spaces" (e.g. simplicial varieties). Then, a deeper theoretical understanding of these motivic categories should shed light on the more concrete problems described above.

Steenrod operations are natural transformations between singular cohomology groups of topological spaces. Similar constructions can be done in algebraic geometry. Such a construction is possible at the level of Chow groups but it is possible to define it on all the motivic cohomology of algebraic varieties (and more generally of all spaces in the context of motivic homotopy). This was used in the proof of Bloch-Kato conjecture but the original construction and study of these operations by Voevodsky contained several gaps that were fixed in [R12] where a "relative" construction of the motivic Steenrod operations is introduced.

4.2.2.6. *Zeta functions and cohomologies for arithmetic schemes*. Another great challenge of modern arithmetic geometry is to construct cohomology theories suitable for the study of zeta functions in characteristic zero. For varieties over finite fields, l-adic and crystalline cohomology provide a spectral description of zeta functions. This is the starting point of the proof of the rationality, the functional equation and the Riemann conjecture in characteristic p. Moreover, Weil-étale motivic cohomology gives a simple description of special values for these zeta functions. It is very likely that this picture can be extended to the category of all arithmetic schemes (i.e. algebraic over the integers), and in particular to the Riemann zeta function. Indeed, on the one hand, Deninger's program describes the cohomology which should be responsible for the general spectral interpretation of zeta functions, and Lichtenbaum conjectured the existence of the Weil-étale cohomology on the category of all arithmetic schemes on the other. There should also exist an intermediate cohomology, called absolute cohomology, which should link Weil-étale to Deninger's conjectural cohomology. Currently, there is no known examples of flat schemes over $\mathbb{Z}$ for which Deninger's cohomology can be constructed, but there exists a conditional definition for the Weil-étale and the absolute cohomology. An important research project aims at developing a motivic formalism underlying the Weil-étale and the absolute cohomology. Such a formalism would provide a powerful tool for the study of (widely open) special values conjectures, such as the BSD conjecture. It would also be a very significant contribution to Deninger's program, since Deninger's cohomology is expected to be a realization functor for these motives.

5. Scientific and technical program, project management

5.1. **Relevance and complementarity of the members in the project.** We present below the researchers involved in the project, insisting more specifically on their mathematical interests and achievements related to it. We then explain the reasons that lead to the constitution of this team.

O. Benoist (CR - Strasbourg). Olivier Benoist's main research topic has been the global geometry of moduli spaces of algebraic varieties. He studied the separetedness and quasi-projectivity of moduli spaces of complete intersections. Using ideas from the MMP, he investigated their birational geometry, with a view to constructing examples of complete families of smooth varieties. In this direction, he constructed the first examples of complete families of non-degenerate smooth space curves, relating this classical question to understanding the behaviour of strong semistability of vector bundles under specialization. He also extended the classical Chevalley-Kleiman projectivity criterion to arbitrary normal varieties. He recently prepared a Bourbaki about the recent progress on the Tate conjecture for K3 surfaces.

A. Cadoret (Prof. Associé - Ecole Polytechnique). A. Cadoret's work focuses on (ℓ-adic and modulo ℓ) representations of the étale fundamental group of schemes with applications to the variation of arithmetico-geometric invariants in families of motives and to ℓ-independence in families of geometric ℓ-adic Galois representations. Jointly with Tamagawa, she showed that the ℓ-adic exceptional locus attached to 1-dimensional families of 'lots of' natural ℓ-adic representations (as those arising from the ℓ-adic cohomology of smooth proper schemes or the universal extension attached to higher ℓ-adic Abel-Jacobi maps) is 'small' (Inventiones Math. [CT12a]), (Duke Math. J. [CT12b]), (Duke Math. J. [CT13a]). They also proved the genus conjecture, the arithmetic gonality conjecture and some cases of the geometric gonality conjecture for abstract modular curves parametrizing torsion in families of modulo ℓ representations of the étale fundamental group of curves in *arbitrary* characteristic. She recently completed the proof of the genus conjecture for arbitrary abstract modular curves (Submitted [CT14a]). She also extended Serre's adelic open image theorem for elliptic curves to arbitrary families of 1-motives ([C15b]) and obtained evidence for the ℓ-independence of the Tate conjecture in characteristic 0 ([C15a]).
In 2013 and 2015, she got four-months invited professor positions at R.I.M.S. (Kyoto-Japan) and, in 2013-14, she visited the I.A.S. (Princeton-U.S.A.) as a Von Neumann fellow. She previously conducted research projects in collaboration with Japan (PHC Sakura, joint CNRS/JSPS) and in the frame of the ANR (AriVAF).

F. Charles (PU - Paris 11). F. Charles works on various aspects of the geometry of algebraic cycles. Some recent work has dealt with motivic conjectures in the case of varieties with vanishing first Chern class. In particular, he has proved Artin's conjecture on supersingular K3 surfaces, thus proving the Tate conjecture for K3 surfaces over finite fields (Inventiones Math. [ChMa13]). In recent work, he has improved on this result by proving a version of Zarhin's trick for K3 surfaces over arbitrary fields, thus giving a new direct proof of the Tate conjecture for K3 surfaces in full generality – improving on previous results that held in odd characteristic and relied on the Kuga-Satake construction – and giving new proofs of results of Zarhin-Skorobogatov. In other papers, he has dealt with varieties over number fields, around specialization problems for Picard groups of K3 surfaces for instance, and the Arakelov geometry of some Hodge loci. In complex geometry, he has proved the standard conjectures for some holomorphic symplectic manifolds, joint with Markman (Compositio Math. [ChMa13]) using arguments from twistor geometry. In joint work with B. Poonen, he has proved Bertini irreducibility theorems over finite fields.
F. Charles' work on the Tate conjecture for K3 surfaces has been recently reviewed – along with other authors' work – in O. Benoist's Bourbaki seminar. He is currently a visiting professor at the Massachussets Institute of Technology. He received the Peccot-Vimont prize of the Collège de France in 2014.

C. Demarche (MCU- Paris 6). C. Demarche studies local-global principles for rational points and zero-cycles of degree one on algebraic varieties, cohomological obstructions to Hasse principle and weak or strong approximation, with emphasis on algebraic groups and their homogeneous spaces. In particular, he obtained comparison results for several cohomological obstructions to Hasse principle for smooth projective varieties (answering a question of Poonen) and he studied the Brauer-Manin obstruction to weak and strong approximation for certain classes of homogeneous spaces for algebraic groups (e.g. joint works with Borovoi and with Wei). He also obtained arithmetic duality theorems for Galois cohomology and, together with Borovoi and Harari, computed unramifed cohomology groups for homogeneous spaces (Ann. Sci. Éc. Norm. Sup. [BDH13]). The techniques he handles include étale cohomology, Brauer group, unramified cohomology, non-abelian cohomology (in the sense of Giraud,

Breen), algebraic groups, group schemes *etc.*
In 2012, he received a Research Award ("Prime d'Excellence Scientifique").

B. Klingler (PU Paris 7/I.U.F.) B. Klingler's work focuses on Shimura varieties and non-abelian Hodge theory. Together with Ullmo and Yafaev, he proved the André-Oort conjecture on the distribution of special points on Shimura varieties under the Generalized Riemann Hypothesis (Annals of Math. [KlY13]), and the hyperbolic Ax-Lindemann-Weierstrass conjecture for the uniformizing map of Shimura varieties (Submitted [KlUY13]). He used non-abelian Hodge theory *à la Simpson* to prove various restrictions on complex representations of the topological fundamental group of a smooth complex projective variety X. In particular, in a recent joint work with Brunebarbe and Totaro, he showed a link between the existence of such representations with infinite image and the existence of algebraic symmetric differentials on X (Duke Math. J. [BrKlT13]).
B. Klingler is a junior member of the I.U.F. since September 2011.

Lie Fu (MCU - Lyon 1) Lie Fu studies algebraic cycles and their intersection theory, especially for holomorphic symplectic varieties. His works are mostly inspired by the Bloch-Beilinson conjecture on Chow rings of complex varieties. On of his results in thesis is to prove the equivariant Bloch conjecture for any polarized symplectic automorphisms of Fano varieties of lines of cubic fourfolds. Recently he proves the Beauville-Voisin conjecture for a series of holomorphic symplectic varieties, namely the generalized Kummer varieties.

B. Morin (CR - Bordeaux). B. Morin studies Weil-étale cohomology, special values of zeta functions and Deninger's program. In a joint work with Flach, he defined a Weil-étale topos for arbitrary regular arithmetic schemes which gives the right definition of the Weil-étale cohomology with $\mathbb{R}$-coefficients. In a recent paper [Duke **163**], a conditional construction of the Weil-étale cohomology with $\mathbb{Z}$-coefficients is given. This yields (unconditionally) the right definition for cellular schemes over number rings and allows a description of zeta-values at $s = 0$, partially proving some conjectures of Lichtenbaum. In a forthcoming joint work with Flach, the same construction is generalized in order to define Weil-étale cohomology with $\mathbb{Z}(n)$-coefficients and absolute cohomology, to prove the corresponding arithmetic duality and to describe zeta-values at any integer argument $s = n$.

F. Orgogozo (CR and PCC - Ecole polytechnique). F. Orgogozo has mostly worked on finiteness problems in étale cohomology, and related topics: the estimation of cohomological dimension of fields (K. Katô's conjecture on p-dimension of fields, in collaboration with O. Gabber (Invent. math., [OG8]), and the commutation with base-change of higher direct images (Deligne's conjecture on vanishing cycles over a high dimensional base). After having proved a uniformity-in-ℓ result for higher direct images (relevant to questions of potential independence of ℓ-adic representations), he positively answered, in a recent joint work with D. Madore ([MO13]), a question of Poonen et al. on the calculability (in the sens of Church-Turing) of the étale cohomology modulo ℓ of varieties over an algebraically closed field of characteristic $\neq \ell$. He handles technics like toposes, étale cohomology, non-abelian cohomology, *etc.* He is coeditor, with L. Illusie and Y. Laszlo, of the Astérisque volume on Gabber's work ([ILO14]). In 2006-2007, he visited Tōkyō University (6 months, invited by T. Saitô) and between 2012 and 2014, he was twice invited at IHÉS (12 months, invited by A. Abbes).

A. Pirutka (CR and PCC - Ecole polytechnique). The research interests of A. Pirutka focus on the study of cycles and rational points on algebraic varieties. She has also worked on rationality properties of algebraic varieties and questions on zero-cycles. She was studying the integral versions of the Tate conjecture and relations with some birational invariants of cohomological nature, coming from Rost cycle modules, this includes in particular the unramified cohomology groups; some of these invariants provided a counterexample for a strong form of the integral Tate conjecture over finite fields (Algebra and Number Theory [Pi11]). In a joint work with N. Yagita, some structural properties of counterexamples are investigated. She also established that the integral Tate conjecture does hold for

cubic fourfolds over a finite field, in a joint work with F. Charles (Compositio Math. [ChPi13]). The methods she uses include some techniques coming from the étale cohomology, K-theory and motivic cohomology, as well as some fine properties of Galois cohomology of function fields, deformation theory and intersection theory for algebraic cycles.
In 2013, she has been invited to the University of Zürich by A. Kresch, for a four-month period.

J. Riou (MCU – Paris 11). In his PhD thesis, J. Riou studied the operations on algebraic K-theory using a strategy based on the homotopy theory of schemes. The idea was to reduce constructions and statements on higher K-theory or other cohomology theories to results on K_0-groups or Chow groups. This led to alternative proofs of some Riemann-Roch type theorems. Some of these results were used by Cisinski and Déglise in their construction of triangulated categories of mixed motives over a general base scheme. He introduced a simple tool to correct the construction and the verification of the properties of the Steenrod operations acting on motivic cohomology. The Steenrod operations are used in the proof of the Bloch-Kato conjecture relating étale cohomology and motivic cohomology. Through his training and the preparation of a Bourbaki talk, J. Riou has become an expert on the various aspects of the proof of the Bloch-Kato conjecture. He was also part of the group who has completed the writing of the proofs of very significant results by Gabber in algebraic geometry and étale cohomology; his contribution has had a focus on cohomological applications: absolute purity in étale cohomology and existence of dualising complexes over general base schemes.

O. Wittenberg (CR - ENS Paris). O Wittenberg has worked on several problems related to zero-cycles on varieties defined over number fields and local fields, among which a conjecture of Colliot-Thélène, Kato, Saito on the existence of global zero-cycles on arbitrary smooth varieties over number fields (Duke Math. J. [Wi12]), and the study of the cycle class map to integral ℓ-adic cohomology for smooth projective surfaces over local fields. A recent joint work with Y. Harpaz settles the above-mentioned conjecture for zero-cycles over number fields in the case of fibrations into rationally connected varieties over a projective space when the fibers satisfy the conjecture. In a recent paper joint with Esnault ([EsWi14]), the cycle class map to integral ℓ-adic cohomology was proved to be injective for a large class of surfaces with positive geometric genus. The method relies on the theorem of Saito and Sato mentioned in (4) (d) and on the theorem of Rapoport-Zink (monodromy-weight conjecture for surfaces) and consists in relating the problem at hand with the integral variant of the Tate conjecture for singular surfaces over finite fields. In another direction, a joint paper with Esnault and Levine (J. Alg. Geom. [EsLWi13]) establishes a relation between the possible degrees of zero-cycles on a variety over a strictly local field and the Euler characteristics of coherent sheaves defined on the given variety. These ideas have played a key role in the proof of a conjecture of Kato and Kuzumaki asserting a variant of the C_2 property for the field of p-adic numbers ([Wi13]).

The researchers listed above fit the necessary high-level scientific criteria of mathematical excellence and consistency of their research area and research plans with those of the project. Above that, their skills and approaches reflect multiple aspects of the investigation of the cohomology of algebraic varieties and its applications, going from the development of conceptual tools to explicit computations. We expect that combining these diverse profiles will generate new ideas leading to breakthroughs in the problems at stake in our project. Another main feature of ECOVA is that it is built on a pre-existing network of mathematical collaborations involving:

(1) Joint collaborations at the national (*e.g.* Charles-Bost/Pacienza, Orgogozo-Madore) and international (*e.g.* Cadoret-Tamagawa, Cadoret-Kret, Charles-Markman/Poonen, Demarche-Borovoi/Wei, Klingler-Totaro/Yafaev, Pirutka-Yagita, Wittenberg-Esnault/Harpaz/Levine/Skorobogatov) levels.

(2) Joint collaborations and activities between the members of the projects:

- Joint papers (*e.g.* [ChPi13]).
- Joint working groups:
  – ‘’Théorèmes de finitude en cohomologie étale (d'après O. Gabber)’ (2008-09, Orgogozo (org.), Riou)
  – ‘Zéro-cycles sur les variétés p-adique’ (2010 - Charles (org.), Pirutka (org.), Wittenberg (org.))
  – ‘Déformations p-adiques des classes des cycles algébriques (d'après S. Bloch, H. Esnault et M. Kerz)’ (2013 - Cadoret (org.), Charles, Klingler, Pirutka (org.), Riou)
- Joint seminars:
  – ‘Autour des cycles algébriques’ (Cadoret, Klingler, Pirutka).
  – ‘Variétés rationnelles’ (Demarche (regularly followed by Pirutka, Wittenberg)).

This should ensure the high capacity of the involved researchers to interact on common mathematical problems in the cohomological study of algebraic varieties.

5.2. **Scientific program - technical description.** The 48 months of the project will be structured around three conference-like events (Opening, mid-term and closing meetings - subsection 5.2.2.3) but the main features of ECOVA is to create and maintain constant interactions between the members of the projects and the members of the projects and outside researchers by a series of more specific regular events (working groups, one-day meetings, invited lectures, seminars - subsections 5.2.2.1) organized all year round as well as by supporting visit to and invitation of outside members, attending conferences (subsection 5.2.2.4). This organisation should provide a constant and uniform progress in the four-year period of our project.

In Subsection 5.2.1, we give a sample of topics for the organizational events in the frame of our project. In selected cases we make precise
- the members of the project whose research interests are close to the topics considered and who will be designed to organize scientifically the events (detailed scientific programs of working groups, invited lectures)
- the relations with the research directions developed in section 4.
- the type of the event (working groups, one-day meetings, invited lectures *etc.*), which seems more appropriate for the goals of the event: from learning new technics to making concrete progress in the frame of our project.

For the remaining proposed topics, we give only general indications. This list is not exhaustive and we plan to develop or modify the program below with respect to our progress as well as new research results which will appear in subjects related to ECOVA.

In Subsection 5.2.2, we give brief technical descriptions of the different type of events and item of expenditure. The financial description itself is postponed to subsection 5.3.

5.2.1. *Proposed topics.* The presentation of the topics we propose below is organized according to the two main directions (Part I and Part II) in Section 4 (in particular, it is not chronological).

Around the Tate conjecture (scientific coordinators: A. Cadoret, F. Charles, A. Pirutka)
The recent developments on the Tate conjecture for $K3$ surfaces have been achieved using various modern tools, such as moduli spaces of sheaves, integral models of Shimura varieties, as well as cristalline methods. In the first part of this working-group we will review classical results; in particular, the proof of the celebrated theorems of Faltings for abelian varieties. The second part will be devoted to a recent approach of Kato, generalizing the one of Faltings and built on the introduction of heights on motives over number fields. The aim of the third part will be to become familiar with the various approaches for the Tate conjecture for $K3$ surfaces.

This topics is closely related to subsection 4.1.1 and the ℓ-independence questions adressed in subsections 4.1.2 and 4.2.1.1.

Part A (learning working group):
(1) Introduction: motivic inspirations, brief overview (various base fields, semi simplicity, fullness, fullness for the power);
(2) ℓ-independence over finite fields;
(3) Divisors on abelian varieties - in particular theorem of Faltings.

Part B (research working group) Heights of motives and the Tate conjecture (after Kato).

Part C (research working group) Developments on $K3$ surfaces:
(1) classical results for elliptic $K3$ surfaces;
(2) integral models of Shimura varieties and finiteness results (after Pera);
(3) finiteness results for moduli spaces of sheaves on K3 surfaces (after Charles);
(4) characteristic p methods and unirationality of supersingular K3 surfaces (after Liedtke [Li13]).

Shimura varieties and Mumford-Tate domains (scientific coordinators: A. Cadoret, B. Klingler)

Since their introduction in the late sixties, Shimura varieties have played an increasing role in many areas of algebraic and arithmetic geometry. This is due to their rich arithmetico-geometric structures (Hecke operators, special subvarieties, integral models and reduction), in turn reflected in the remarkable properties of their cohomology, and to their modular interpretations. They are related to certain variations of Hodge structures and, in some case, are parameters spaces for abelian varieties and $K3$-surfaces. Their ℓ-adic cohomology provides a realm of examples for the Langlands' conjectures and the Tate conjecture. The aim of this series of meetings on Shimura varieties is to become familiar with the standard theory, adquire advanced tools that might be useful to study specific questions related to our research work and understand their part in area closely related to the subject.

Shimura varieties are ubiquitous in the study of the problems discussed in subsections 4.1.1, 4.1.2 and 4.2.1.1.

Part A (learning working group) Standard theory: construction, complex structure, canonical models, modular interpretations, good reduction (after Deligne, Milne).

Part B - Advanced topics:

(1) (research working group and invited lectures) Study of special sub-varieties, equidistribution, André-Oort conjecture (after Clozel, Klingler, Ullmo, Yafaev). Suggested speakers: C. Daw, Z. Gao, A. Kret, M. Orr.
(2) (invited lectures) Integral models (after Kisin) and application to the Mumford-Tate conjecture. Suggested speakers: C. Cornut, B. Moonen.
(3) (invited lectures) Mumford-Tate domains (along the book of Green-Griffiths-Kerr), applications to the general study of the Hodge locus.
(4) (invited lectures) Introduction to the Langlands program and ℓ-adic cohomology of Shimura varieties. Suggested speaker: G. Chenevier.
(5) (invited lectures) Applications of weight structures on mixed motives to the study of Shimura varieties. Suggested speaker: J. Wildeshaus.

Representations of étale fundamental groups over finite fields (scientific coordinators: A. Cadoret, F. Orgogozo)

Representations of étale fundamental groups naturally arise in the study of the variation of étale cohomology (with ℓ-adic or finite coefficients) in the fibers of a smooth proper morphism. By means of the specialization theory for étale fundamental group, it is often possible to reduce problems to the case where the base field is finite. It is thus of primary importance to understand this situation, which has been thoroughly studied over more than fifty years with emblematic results as the proofs of the Weil conjectures by Deligne, pioneering works of Katz on rigid systems and, recently, the breathtaking results achieved - among others - by L. Lafforgue and V. Lafforgue in the frame of the geometric Langlands' correspondence. This series of meeting will review as thoroughly as possible the results alluded to above and some refinements such as ℓ-independency and counting by Chin and Deligne.

This topics is at the core of the ℓ-indepency questions developed in subsections **??** and 4.2.1.1.

Part A: (learning working groups) The Weil and monodromy-weight conjectures:

(1) Review of Weil's conjectures (after Deligne - Weil 1, Weil 2). Applications.
(2) Monodromy-weight conjecture:
   (a) Statement;
   (b) Over finite field (after Deligne);
   (c) Partial results in characteristic 0 (after Scholze). This part of the working group require an introduction to perfectoid spaces and could possibly be organized in parallel with the second part of the suggested topics 'Recent developments on p-adic comparison theorems' described below.

Part B: Introduction to the geometric Langlands correspondence and derived results:

(1) (Invited lectures) Introduction to Langlands' correspondence (works of L. Lafforgue and V. Lafforgue). Suggested speakers: J. Heinloth, V. Lafforgue, S. Morel, Ngo Dac Tuan.
(2) (research working group) Advanced results based on Langlands' correspondence:
   (a) ℓ-independence [Chi04];
   (b) complexity of ℓ-adic sheaves; applications ([FKM13]) (Prerequisite for (c));
   (c) finiteness [D12] and [EsK12];
   (d) application to inverse Galois theory.

Part C: (invited lectures and research working group) Works of Katz.

Non-abelian Hodge theory (scientific coordinator: B. Klingler)

(1) (research working group and invited lectures) Standard theory over $\mathbb{C}$: Narasimhan-Seshadri theorem, Hitchin-Simpson's theory [Si91].
(2) (research working group) Non-abelian Hodge theory in positive characteristic: the work of Ogus-Vologodsky [OV07].
(3) (research working group and invited lectures) p-adic non-abelian Hodge theory: Faltings' approach [Fa05], Deninger and Werner's work [DenW05], Abbes and Gros' approach [AG11]. Invited lectures. Suggested speaker: A. Abbes.

Recent developments on p-adic comparison theorems (scientific coordinator: O. Wittenberg) (research working group), two possible directions:

(1) Beilinson's new proof of the B_{dR} conjecture ([Be12]), and the subsequent new proofs of the Fontaine–Jannsen B_{st} conjecture ([Bh12], [Be13]), via derived de Rham interpretations of the period rings.
(2) Scholze's extension of the de Rham comparison theorem to rigid-analytic varieties, via perfectoid spaces and the pro-etale topology ([S13]).

Computational aspects of étale cohomology (scientific coordinator: F. Orgogozo)

(1) (Learning working group) Survey of point-counting techniques and results [Se12].
(2) (Invited lectures) Couveignes and Edixhoven's work on Ramanujan's τ [EC11]. Suggested speaker: Bas Edixhoven.
(3) (Invited lectures) Works of C. Faber et al. on moduli spaces of curves and relation with motives and automorphic forms. Suggested speaker: G. Chenevier.

Zero-cycles : classical results, recent developments and perspectives (working group, scientific coordinators: C. Demarche, A. Pirutka, O. Wittenberg)

Cohomological methods are of particular efficiency in the study of zero-cycles, be it structural questions for the group $CH_0(X)$ or local-global principles over various fields (works of Saito and Sato, Esnault and Wittenberg, Harpaz and Wittenberg, see section 4.2.2.3). On the other hand, the properties of this group encode an important information about the variety itself, in particular through a concept of 'CH_0-dimension' (already in the work of Bloch and Srinivas [BS83], see [CTPi14] for a recent application).

This topics are related to some concrete problems on zero-cycles in the frame of our project (see sections 4.2.2.3, 4.2.2.4), which we recall now:
- a conjecture of Colliot-Thélène, Kato and Saito: on any smooth proper variety over a number field, the existence of a zero-cycle of degree one should be equivalent to the existence of a collection of local zero-cycles of degree one such that their cycle classes come from a single global étale cohomology class (recent progress by Y. Harpaz and O. Wittenberg);
- local-global principles for varieties fibred over a curve over a finite field, computations of the unramified cohomology groups as a way to inderstanf these principles, developping other invariants.

The goal of the working group on zero-cycles is to make progress on question above and to encourage collaborations between the membres of the group working in this area (C. Demarche, A. Pirutka, O. Wittenberg) as well as international experts (Y. Harpaz, H. Esnault). To start with, we will also organize a learning working group for young researchers to provide an introduction to this subject.

(1) (learning working group) classical results: representability of CH_0 over $\mathbb{C}$, Bloch's conjecture, theorem of Mumford, decomposition of the diagonal revisited;

(2) (research working group) structural properties over various "arithmetical fields" and local-global principles.

Algebraic cycles and algebraic groups, motivic theory and Chow groups (scientific coordinators: C. Demarche, A. Pirutka, J. Riou)

Algebraic groups and their classifying spaces provide a large field of investigation in the context of the study of algebraic cycles, in particular, they are an essential tool for constructing counterexamples for the integral Hodge and Tate conjectures, as recalled in section 4.2.2.1. Classifying spaces are naturally objects of a motivic category and their properties are also important in the contruction of Steenrod operations and in the Bloch-Kato conjecture (cf. section 4.2.2.5).
As a first part of a working group on the questions above, we propose a learning introductory working group on these mordern tools. The second part of the working group will be devoted to active research questions, in particular to the study of cohomological invariants of algebraic groups (such as Rost invariant). These invariants are in general difficult to construct and to compute, they are naturally of interest for us, in particular, with relation to a recent work of Merkurjev, where motivic cohomology methods allow to relate them to the study of Chow groups. In September 2015 there will be a conference in C.I.R.M dedicated to A. Merkurjev, so that we hope to have an opportunity to invite

international experts in this area.

(1) Classifying spaces, Chow groups of BG. Relation to motivic cohomology and operations. Learning working group and invited lectures. Suggested speakers: J. Riou, B. Kahn.
(2) Cohomological invariants of algebraic groups. Case of exceptional groups. Relation to $CH^2(BG)$. Invited lectures. Suggested speakers: Ph. Gille, S. Garibaldi, A. Merkurjev.

As already mentioned, the above list in not exhaustive and may be adjusted in the course of the project. Other suggested topics include the study of periods (after Ayoub, Brown), Height on Chow groups (after Beilinson, Bloch, Gillet-Soulé *et al.*) *etc.*

5.2.2. *Organizational events - technical description.*

5.2.2.1. *Regular meetings*. We provide below a technical description of the regular meetings we plan to organize during the 48 months of the projects. These should take place in one of the participating institutions (Paris 6, 7, 11, E.P. or E.N.S.) or at I.H.P. or I.H.E.S.

Weekly or monthly working groups. Weekly or monthly working groups are an opportunity for researchers interested in the same topics to meet regularly in order to discuss it thoroughly. Due to the high involvement they require (both in the elaboration of the general scientific program and preparation of the talks), they are an efficient way to learn new technics and initiate collaborations. The topics covered could be learning a general theory or reading one (or a series of) research article(s) introducing important technics and results. In the proposed topics above, we refer to the former as 'learning' working groups and to the later as 'research' working groups. The learning working groups should be accessible to a rather broad audience including graduate students in algebraic and arithmetic geometry. In order to interact actively with other researchers in the area, we expect to draw attention of wide audience, young researchers and doctoral fellows for learning workshops, as well as to invite outside experts for research workshops to enrich understanding of the subjects which is less familiar for us. We plan that at least half of the talks will be given by doctoral fellows or outside researchers. The format may depend on the topics but, on the average, we plan to organize one long or two-to-three shorter working groups per year with a two-hours meeting every week from late September to late April.

Invited lectures. The purpose of invited lectures will be to introduce an advanced research topics and accompanying technics in a series of lectures designed for doctoral fellows, post-doctoral fellows and researchers who are non-specialists but may need to learn these technics for their own research purposes or understand how the research area covered by the lectures might be related to theirs. Again, the number of lectures may depend on the topics but a typical format should be around 12 hours over two weeks (one two-hours class every two days). These could be organized as independent events or as events complimentary or preliminary to a working group. Also, the lectures themselves could be given by a single speaker or by two or three collaborating speakers (involving some of the members of the project).

One-day meetings. These meetings will consist in three research talks on a common topics and be labelled ECOVA (by opposition to regular monthly seminars organized according to the same principle - see Subsection 5.2.2.2). They could be organized on an average of twice a year, possibly in parallel with invited lectures (typically, speakers could be the invited lecturer, a member of ECOVA and a third invited speaker).

5.2.2.2. *Others*. Some of the members of the projects are involved in the organization of regular monthly seminars : 'Cycles Algébriques' - Paris 6, 7, E.P. (A. Cadoret, B. Klingler, A. Pirutka), 'Variétés rationelles' - I.H.P. (C. Demarche), 'Séminaire de géométrie arithmétique Paris-Pekin-Tokyo' - I.H.E.S. (F. Orgogozo). Though ECOVA is not designed for funding these seminars, the involvements of some of its members in their organization should imply naturally complementary interaction. Typically, researchers invited in the frame of ECOVA (for research collaboration, invited lectures *etc*) may naturally be offered to give a research talk in one of these seminars.

5.2.2.3. *Opening, mid-term and closing meetings*.

Opening meeting. The purpose of this short three-days meeting is to initiate formally the project. It will be mostly devoted to internal exchanges, in particular each member will present his/her recent research achievements and research projects and explain how they are related to the other members' ones. Some time will also be devoted to preparing the mathematical and technical organization (elaboration of the mathematical program for 2014-15, preliminary discussions about the mid-term meeting *etc*). The meeting (except internal discussions) will be open to the public but, *a priori*, presentations will be only given by the members. Depending on room available, we will organize it at I.H.P. or one of the participating institutions.

The aim of the mid-term and closing meetings is to draw the attention of the mathematical community - from graduate students to international specialists - on the topics and achievements of the project. It will also be the occasion to gather together the leading world specialists in the fields covered by ECOVA.

Mid-term meeting. The mid-term meeting will be the central event of the project. We plan to organize it during the third year of the project (early or late summer). It should consists of a 5-days mixed summer school/research conference with (three to five) series of invited lectures of graduate and post-graduate level, short talks by young researchers and a few longer talks by leading specialists. We expect a large audience hence we plan to organize this meeting either in C.I.R.M. or I.H.P.

Closing meeting. This meeting should be a standard 5-days international research conference with one hour talks, 4 to 5 talks/day. We will organize it during the last year of the project. Its primary purpose will be to present the main mathematical achievements obtained in the cohomological study of algebraic varieties during the four years of the project. An average of a third of the talks will be given by the members of ECOVA. We expect a rather large audience hence we plan to organize this meeting either in C.I.R.M. or I.H.P.

5.2.2.4. *Visits, invitations, travellings*. The members of ECOVA are already involved in several joint collaborations at the national (*e.g.* Charles-Bost/Pacienza, Orgogozo-Madore) and international (*e.g.* Cadoret-Tamagawa, Charles-Markman/Poonen, Demarche-Borovoi/Wei, Klingler-Totaro/Yafaev, Pirutka-Yagita, Wittenberg-Esnault/Harpaz/Levine/Skorobogatov) levels. And there is no doubt that they will initiate and develop further collaborations during the 48 months of the project. As a result, a particular emphasis will be put on supporting these collaborations through visits to or invitations of collaborators and attending conferences or special semesters.

5.3. **Financial description.**

5.3.1. *Summary.*

Item of Expenditure	Requested budget
Organizational events (except meetings)	K€34,5 (including adjustment: K€2)
Meetings	
Opening meeting	K€2
Mid-term meeting	K€11,5 (including adjustment: K€1)
Closing meeting	K€11,5 (including adjustment: K€1)
Missions/invitations	K€80,5
Others (computers, books)	K€10

The total requested budget is thus: K€150.

5.3.2. *Justification of requested budget.* The requested budget was estimated on the following basis concerning invitations:

Per-diem (accommodation and living expenses): €100/day
Travels (including connections to/from Airport *etc.*): €200 inside France
€500 outside France

Organizational events (except meetings):

Working groups: Travel expenses for one colleague in France
Number of meetings per year: 20.
Estimated amount: €200x20x4=K€16

Invited lectures (on the basis of 13 days for the invited speaker):
Number of invited lectures per year: 2 but we expect one to be founded by an invited professor position.
Estimated amount: K€1,8x4=K€7,2

One day meetings (2 days for one outside (foreign) speaker not in Paris at the time of the meeting):
Number of meetings per year: 2.
Estimated amount: €700x2x8=K€11,2

Meetings: The amounts '$+x$' refer to small adjustment budget.

- Opening meeting: +K€2

- Mid-term meeting (on the basis of 5 days):

Estimated numbers of funded participants	:	7 (France) ,7 (Foreign)
Estimated amount	:	700x7+1000x7=10,5
Adjustement		+1
	=	K€11,5

- Closing meeting (on the basis of 5 days):

Estimated numbers of funded participants	:	7 (France), 7 (Foreign)
Estimated amount	:	700x7+1000x7=10,5
Adjustement		+1
	=	K€11,5

The above budget are estimations of the contribution of ECOVA. Depending on the place where the meeting are organized (for instance, CIRM), we hope to be able to fund more participants, in particular Ph.D. students and post-doctoral fellows. If necessary, we will also look for additional fundings from other sources (CNRS, participating institutions *etc.*)

Missions/invitations and others: In order to cover visits to or invitations of foreign collaborators, attending conferences, buying computers or books *etc.* we roughly request K€10,5 per members over the 48 months of the project and adjust this amount according to the percentage of implication, obtaining K€90,5 (missions/invitations: K€80,5, others (computers. books): K€10). This budget should be completed by other sources like:

- Invited professors positions both in foreign research institutes for the members of the projects and in the institutes involved in ECOVA for invited lecturer or collaborators.
- Foreign collaborators's grants.
- Outside fundings from conferences, special research semesters *etc.* attended by the members.

6. Data management, data sharing, intellectual property and results exploitation

6.1. **Publications, external communications.** In pure mathematics, one disposes of the following valorization criteria:
- Publication of research papers in international journals or conference proceedings with reading committees;
- Invited Lectures in international research conference or seminars;
- Invited Lectures in research schools or research survey seminars.

A main purpose of ECOVA is to reinforce mathematical exchanges among a small group of young researchers *via* weekly or monthly working groups and to provide them with a regular financial support so that they can visit or invite (potential) collaborators and attend international conferences to communicate their results and initiate new collaborations.

We expect that this strategy will lead to mathematical results fitting with the highest international standards and giving raise to publications and communications as above.

We intend to mention the support of the A.N.R. *via* ECOVA in all the publications written during the project and mathematically related to it. We also expect the researchers involved in the project to acknowledge the support of the A.N.R. *via* ECOVA on their professional webpage, on slides when they give talks supported by the project *etc.*

6.2. **Organizational events.** Also, we expect the organizational events scheduled in the frame of the project to draw the attention of the mathematical community - from graduate students to international specialists - on the topics and achievements of the project. More precisely, one can distinguish three kinds of events with slightly different audience:

- Local events: Opening meeting, regular (weekly, monthly) events (working groups, one-day meetings) and invited lectures by specialists. The opening meeting will contain the presentation by each member of his/her research project and its correlations with the other members'. It will be open to the public but, *a priori*, presentations will be only given by the members and the purpose of this meeting will be, above all, to initiate the project. The regular events will be organized according to the needs of the members of the project (see §5) but will be advertised and open to public and at least half of the talks should be presented by researchers not involved in the project.

- National and international events. These are the mid-project mixed summer school/research conference and final research conference, for which we expect a national and international audience. Let us also mention that in case closely related to ECOVA conferences happen to be organized during the project but not directly in its frame, ECOVA may fund it partially provided the conference committees acknowledge its contribution and the scientific committee involves at least one of the member of ECOVA.

We intend to advertise these events both at the local, national (one day meeting, working groups, invited lectures) and international (mid-project mixed summer school/research conference, final research conference) level *via* the usual processes: creation of a mailing lists, diffusion on pre-existing mailing lists, sending posters to laboratory *etc.*.

6.3. **Webpage.** We intend to create a webpage devoted to the project and containing:

- A brief description of the project;
- A list of the members and links to their professional webpages;
- A calendar of the events scheduled in the frame of the projects with additional specific information (practical informations, scientific program, lecture notes, suggested pre-requisites *etc.*)
- A series of links to other mathematical events (conferences, series of lectures *etc*) or activities (seminars *etc*) supported or related to the project.

6.4. **Graduate to post-doctoral training.** Let us also mention that the members of the project all belong to laboratories involved in graduate, doctoral and post-doctoral training (Master AAG in Paris 11 and E.P., Joint Master in Paris 6, Paris 7 and E.N.S., graduate schools, Masters in Bordeaux, lyon and Strasbourg), and have the opportunity to supervise Master 2 and Ph.D. thesis or to teach at the graduate or post-doctoral level in this frame. This will be one more opportunity to attract graduate students and young researchers to the topics of ECOVA. In case of young motivated researchers, for instance a Ph.D. student starting a thesis under the supervision of one of the member of the project, ECOVA could be used as a financial complementary to other fundings (from doctoral schools, travel grants *etc.*) to support his/her attending research schools or conferences.

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