20/ may choose $n_0 = p$ prime

if $m \wedge p = 1$ ~~then m^a is non-zero modulo p for all a hence $m^a - m^{a'}$ is divisible by p for some $a' > a$.~~

m is of finite order in $\mathbb{F}_p^\times$.

$\exists a \geq 1$ $\quad p \mid m^a - 1$ $\quad$ i.e. $|m^a - 1| < 1$

$\Rightarrow |m|^a = 1$. ///

<u>Notations</u>

$M_{\mathbb{Q}} = \{\infty\} \cup \{\text{primes}\} \xleftrightarrow{1:1}$ non trivial norms on $\mathbb{Q}$ modulo the equivalence relation

$|.|_1 \sim |.|_2$ iff $|.|_1^{\alpha} = |.|_2^{\beta}$ $\exists \alpha, \beta > 0$.

$v \in M_{\mathbb{Q}} \curvearrowright |.|_v$ the associated norm

abuse of notation identifies v and $|.|_v$.

<u>Aim</u> = extends this correspondence to any number field K

$$M_K = \{ \text{multiplicative norms on } K \text{ s.t. } |.|\big|_{\mathbb{Q}} = |.|_v,\ v \in M_{\mathbb{Q}} \} \xrightarrow{\text{rest}} M_{\mathbb{Q}}.$$

- the archimedean case

$K = \mathbb{Q}[T]/_P$ $\quad P \in \mathbb{Z}[T]$ irreducible $/\mathbb{Q}$.

$$\text{Zeroes in } \mathbb{C} \text{ of } P/\mathbb{Q} = \{ \underbrace{x_1, \dots, x_r}_{\in \mathbb{R}}, \underbrace{y_1, \overline{y_1}, \dots, y_s, \overline{y_s}}_{\in \mathbb{C} \setminus \mathbb{R}} \}$$

<u>Embeddings</u>

$1 \leq i \leq r$ $\quad \sigma_i : K \hookrightarrow \mathbb{R}$ $\quad \sigma_i(T) = x_i$.

$x \in K$ $\quad |x|_{\sigma_i}^{\mathbb{R}} := |\sigma_i(x)|_{\infty}^{\mathbb{R}}$.

$1 \leq j \leq s$ $\quad \tau_j : K \hookrightarrow \mathbb{C}$ $\quad |x|_{j,\mathbb{C}} = |\tau_j(x)|_{\infty}$.