2/

$$M_{K,\infty} = \{ |\cdot| \text{ norms on } K \text{ s.t. } |\cdot|\big|_{\mathbb{Q}} = |\cdot|_\infty \}$$

<u>Thm</u> $M_{K,\infty} = \{ |\cdot|_{i,\mathbb{R}} \}_{i=1}^{r} \cup \{ |\cdot|_{j,\mathbb{C}} \}_{j=1}^{s}$

$\mathrm{Card}\,(M_{K,\infty}) = r + s \leq r + 2s = [K : \mathbb{Q}]$.

$v \in M_{K,\infty}$ set $n_v = [\underset{\substack{\uparrow \\ \text{completion of } K \text{ wrt } |\cdot|_v}}{K_v} : \mathbb{Q}_v] = \begin{cases} 1 & \text{if } v = |\cdot|_{i,\mathbb{R}} \\ 2 & \text{if } v = |\cdot|_{j,\mathbb{C}} \end{cases}$

<u>Fact</u>: $x \in K$ $\quad \prod_{v \in M_{K,\infty}} |x|_v^{n_v} = |N_{K/\mathbb{Q}}(x)|_\infty$

<u>proof</u>

$$\prod |x|_v^{n_v} = \prod_{i=1}^{r} |\sigma_i(x)|_\infty \times \prod_{j=1}^{s} \underbrace{|\tau_j(x)|^2}_{= |\tau_j(x)| \times |\overline{\tau_j}(x)|}$$

$$\mathrm{Gal}(\mathbb{C}/\mathbb{Q}) \cdot x = \{ \sigma_i(x), \tau_j(x), \overline{\tau_j}(x) \}.$$

///

<u>proof</u> - $f \in M_{K,\infty}$ $\quad f : K \to \mathbb{R}_+$ norm $f\big|_{\mathbb{Q}} = |\cdot|_\infty$.

$\hat{K}$ = completion w.r.t f. $\supseteq$ closure of $\mathbb{Q}$ $\to$ completion of $(\mathbb{Q}, |\cdot|_\infty) = (\mathbb{R}, |\cdot|_\infty)$

Gelfand $\quad \hat{K} = (\mathbb{R}, |\cdot|_\infty)$ or $= (\mathbb{C}, |\cdot|_\infty)$.

$K = \mathbb{Q}[T]/(P) \xhookrightarrow{\mu} \mathbb{C}$ sends T to a root of P in $\hat{K}$.

$f(x) = |\mu(x)|_\infty \qquad \{x_i, y_j\}$